# An FDTD Formulation for Communication Through a Plasma With a Linear Density Profile

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Abstract—A finite-difference time-domain (FDTD) formulation employing an nonlinear bulk velocity of electron is studied and applied to analysis of multiple electromagnetic wave propagation in an unmagnetized and cold plasma layer with a linearly increasing electron density profile. The formulation is suitable for studying the performance of communication through plasma sheath in re-entry or hypersonic vehicle. Resonant absorption of electromagnetic wave and its angle dependency in plasma sheath are analyzed using the developed formulation. Gradient pressure of linear bulk velocity of the electron and distortion of the electron density are applied to the nonlinear correction of electromagnetic wave propagation. The nonlinear interaction of two incident waves and scattering process are investigated. Scattering wave captured at the vehicle shows that high frequency pump wave launched at the vehicle is modulated by low frequency signal wave transmitted from ground. The result is analogous to the Raman scattering process and can be applied to show the feasibility of communication through high density plasma sheath.

# Keywords— Finite-difference time-domain (FDTD), re-entry plasma, cold plasma, linear density profile, Raman scattering

# I. INTRODUCTION

Among the plasma related electromagnetic problems, communication black out during a hypersonic flight or reentry of a vehicle has been an important problem in aerospace engineering society [1]. It is well known that the major cause of communication black out is high density plasma layer formed near surface of the vehicle when the vehicle fly with very high speed or emit its rocket plume [2]. Recently, it is investigated that oscillation of perturbed charge formed inside the high density plasma layer due to resonance absorption can be utilized as an antenna itself [3]. A wave with high power and high frequency radiated from the vehicle scatters at the resonance layer and be modulated by the signal wave launched from the ground.

In this paper, a finite-difference time-domain (FDTD) formulation employing the nonlinear bulk velocity due to the

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Fig. 1. Schematic of multiple electromagnetic wave interaction in plasma sheath with a linear electron density profile

gradient pressure of the linear bulk velocity is studied and applied to multiple electromagnetic wave propagation in an unmagnetized and cold plasma layer with a linearly increasing electron density profile. Nonlinear interaction of two incident waves and scattering process are considered. The scattered wave captured at the vehicle shows that high frequency pump wave is modulated by low frequency signal wave transmitted from the ground.

#### II. FORMULATION

We assume the plasma is cold and unmagnetized, also the motion of ions and neutrals are negligible. Then only a motion of the electron is important in behavior of electromagnetic wave propagation in the plasma. In above assumptions, the Maxwell's equations, the Lorentz's equations of force, the nonlinear bulk velocity and the charge conservation law are

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given to be solved [3]. To formulate the Maxwell's equations in the Yee grid, the E-J collocated scheme is adapted to natural coupling of currents to the electric field in Ampere's law and the nodes for the electron density are located at vertices of the Yee grid as described in [4]. For the sake of simplicity and comparison to the existing analytic solutions, we only consider the 2-D transverse magnetic (TM) mode electromagnetic wave propagation.

## III. NUMERICAL RESULTS

As a validation test, the resonant absorption of TM mode electromagnetic wave is investigated using the developed FDTD formulation. It is well known that normally incident TM mode electromagnetic wave is all reflected at the resonance layer, i.e.,  $z = z_{Res}$ , where the plasma frequency is same as the incident wave frequency [5]. The oblique incident wave, however, reflects at the turning point of plasma, i.e.,  $z = z_T$ , where the plasma frequency corresponds to the value  $\omega_p = \omega_s \cos \theta$ , and converts a part of its energy to electrostatic oscillation in cold plasma layer with a linear electron density.  $\omega_s$  and  $\theta$  are angular frequency and angle of the obliquely incident wave, respectively. The maximum electron density is  $n_0 = 1.0 \times 10^{18} m^{-1}$  and the effective electron collision frequency is  $v = 1.0 \times 10^8 Hz$ . The depth of plasma layer is  $z_{veh} = 1 m$ . The time step value for simulation is  $\Delta t =$ 2.5 ps and the width of each cell is 1.5 mm long.

The validations for the resonant absorption are performed with normal and oblique ( $\theta = 30$ ) angles. The magnitude result of electric field component Ez which is parallel to the direction of density gradient of plasma layer is described for each case in Fig. 2. The resonant absorption is shown only in oblique incident case as expected.

With the same geometry and plasma regime mentioned above, two incident waves are considered in the FDTD simulation. We refer the secondary incident plane wave with high frequency *the pump wave* ( $\vec{k}_{pump}$ ). The pump wave with incident angle  $\psi = 0^{\circ}$  is launched from the vehicle as described in Fig. 1. The fast Fourier transform (FFT) result of the magnetic field of scattering wave at the vehicle, i.e.,  $z = z_R$ , is described in Fig. 3. The result was normalized to its maximum value. It is shown that the scattering modulated by the signal wave has the frequency corresponding to difference of the two incident waves, i.e.,  $f_{scat} = 13 GHz$ .

### IV. CONCLUSION

In this paper, an FDTD formulation for an unmagnetized cold plasma layer with a linear density profile is investigated. The gradient pressure and time dependent perturbation of the electron are employed with the Maxwell's equation. The scattering wave captured at the vehicle has the frequency corresponding to the frequency difference between the signal wave and the pump wave. It is because the pump wave has a



Fig. 2. Resonant absorptions for (a) normal incident and (b) oblique incident



Fig. 3. The normalized amplitude of FFT for Hx component of scattering wave

nonlinear interaction with the perturbed charge at resonance layer due to resonance absorption. The results indicate that it is feasible to simulate the communication through high density plasma layer with developed FDTD formulation.

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