Enhancement of Wireless Power Transfer Efficiency Using Higher Order Spherical Modes

Yoon Goo Kim · Jongmin Park · Sangwook Nam

Abstract

We derive the Z-parameters for the two coupled antennas used for wireless power transfer under the assumption that the antennas are canonical minimum scattering antennas. Using the Z-parameter and the maximum power transfer efficiency formula, we determine the maximum power transfer efficiency of wireless power transfer systems. The results showed that the maximum power transfer efficiency increases as the mode number or the radiation efficiency increases. To verify the theory, we fabricate and measure two different power transfer systems: one comprises two antennas generating TM_{01} mode; the other comprises two antennas generating TM_{02} mode. When the distance between the centers of the antennas was 30 cm, the maximum power transfer efficiency of the antennas generating the TM_{02} mode increased by 62 % compared to that of the antennas generating the TM_{01} mode.

Key words: Canonical Minimum Scattering Antenna, Higher Order Mode, Spherical Mode, Wireless Power Transmission.

I. Introduction

Near-field wireless power transfer has been receiving extensive interest recently, and it is now being widely studied. A near-field wireless power transfer system can be viewed as a coupled antenna system in a near-field region. Hence, the field pattern that an antenna generates has a significant effect on the behavior of the wireless power transfer system. Many researchers, however, have tried to transmit power wirelessly in a near-field region using antennas that generate a mostly fundamental spherical mode $[1] \sim [5]$. In this paper, we attempt to determine a field pattern that is more efficient for wireless power transfer than that of the fundamental mode. The results of our theoretical investigation showed that antennas generating higher-order spherical modes are more efficient for transferring power wirelessly unless the radiation efficiency is too low. These results were verified by an experiment.

II. Mutual Coupling between Two Antennas

We determine the mutual coupling between two antennas under the assumption that the antennas are canonical minimum scattering antennas. The canonical minimum scattering (CMS) antenna does not scatter electromagnetic fields when its local port is open-circuited [6]. Many antennas that are small, relative to wavelength, can be modeled as minimum scattering antennas [7].

The Z-parameter of two coupled CMS antennas can be derived using the method described in [8]. To determine the Z-parameter, we first represent the fields that antennas generate as a superposition of spherical modes. The spherical mode functions and ordering of modes adopted in this paper are the same as those used in the EM simulator FEKO [9]. Let a reciprocal and matched CMS antenna be located on the origin of coordinate system 1 and an identical antenna be located on the origin of coordinate system 2, as shown in Fig. 1. Let the modal transmitting pattern [8] of the antennas be **T**, the modal receiving pattern [8] of the antennas be **R**, and the input impedance of the antennas be Z_{in} . The Z-parameter between the two identical matched and reciprocal CMS antennas is then as follows:

$$Z_{11} = Z_{22} = Z_{in} \tag{1a}$$

$$Z_{12} = Z_{21} = Z_{in} \mathbf{RGT}$$
, (1b)

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Fig. 1. Coordinate systems and antennas. Coordinate system 2 (x_2 , y_2 , z_2 axis) is obtained by translating coordinate system 1 (x_1 , y_1 , z_1 axis). The position of the origin of coordinate system 2 is (r, θ , ϕ) in the spherical coordinate with respect to coordinate system 1.

where the element in the *i*th row and *j*th column of **G** is defined by

$$G_{ij} = \begin{cases} \sqrt{\frac{\nu(\nu+1)}{n(n+1)}} A_{\nu\mu,nm}(r,\theta,\phi) & \text{when } s = \sigma \\ \sqrt{\frac{\nu(\nu+1)}{n(n+1)}} B_{\nu\mu,nm}(r,\theta,\phi) & \text{when } s \neq \sigma, \end{cases}$$
(2)

where $A_{\mu,nm}(r,\theta,\phi)$ and $B_{\nu\mu,nm}(r,\theta,\phi)$ are the functions of the addition theorem in [10]. Herein (s, m, n) and (σ, μ, ν) are the mode indices, and $i=2\{\nu(\nu+1)+\mu-1\}+\sigma$ and $j=2\{n(n+1)+m-1\}+s$. $s,\sigma=1$ denotes TE mode and $s,\sigma=2$ indicates TM mode. *m* is the integer between -n and n, μ is the integer between $-\nu$ and ν , and *n* and ν are positive integers. The coefficient $\sqrt{\nu(\nu+1)/n(n+1)}$ is followed by $A_{\nu\mu,nm}(r,\theta,\phi)$ and $B_{\nu\mu,nm}(r,\theta,\phi)$ because the mode functions used in this paper and the mode functions used in [10] are different.

The power transfer efficiency is defined as the power dissipated at the load in the receiving antenna divided by the power accepted by the transmitting antenna. If the Z-parameter of two coupled antennas is given, then the maximum power transfer efficiency can be calculated by the following equation [11]:

$$PTE^{max} = \frac{|X|^2}{2 - \text{Re}(X^2) + \sqrt{4 - 4\text{Re}(X^2) - \left[\text{Im}(X^2)\right]^2}},$$
 (3)

where $X = Z_{21} / \text{Re}(Z_{11})$.

III. Enhancement of Maximum Power Transfer Efficiency Using Higher Order Spherical Modes

It might be worth investigating the effect of higher-

order modes on wireless power transfer. Intuitively, one would expect that the higher-order mode would be efficient for wireless power transfer. Electric and magnetic energy stored outside a sphere surrounding an antenna and the quality factor (Q) increase as the mode number increases [12]. Therefore, the higher-order mode may be able to increase the coupling coefficient. According to [13], a wireless power transfer system with a large coupling coefficient and large Q is efficient for wireless power transfer.

We examine whether the higher-order mode is indeed efficient for wireless power transfer. We assume that the antennas are CMS antennas and generate only one TE_{0n} (TM_{0n}) mode for simplicity. Using (1), (2), and (2.107) in [14], X in (3) then becomes

$$X = \eta_{rad} A_{n0,n0}(r,\theta,\phi)$$
⁽⁴⁾



Fig. 2. Maximum power transfer efficiencies for antennas generating the TE_{0n} (TM_{0n}) mode when the radiation efficiencies of the antennas are 1.



Fig. 3. Maximum power transfer efficiencies for antennas generating the TE_{01} (TM_{01}) mode and for antennas generating the TE_{02} (TM_{02}) mode for various radiation efficiencies. $\theta = 0$, $\phi = 0$.

where η_{rad} is the radiation efficiency of the antenna. From (3), we find that the maximum power transfer efficiency increases as the absolute value of the imaginary part of *X* or the absolute value of the real part of *X* increases. Because the magnitude of the imaginary part of $A_{n0,n0}(r,\theta,\phi)$ is bigger for larger values of *n*, a mode with large *n* can be efficient for a wireless power transfer. Fig. 2 shows the maximum power transfer efficiency of a wireless power transfer system comprised of two lossless antennas generating TE_{0n} (TM_{0n}) mode against the distance between the antennas. Fig. 2 shows that the maximum power transfer efficiency increases as the mode number increases.

The maximum power transfer efficiency increases with the radiation efficiency since the greater the radiation efficiency, the larger the magnitude of X. Fig. 3 shows the maximum power transfer efficiencies of the wireless power transfer system using the TE_{01} (TM_{01}) and the wireless power transfer system using the TE_{02} (TM_{02}) mode for various radiation efficiencies. It should be noted that the higher-order mode antenna is more efficient than the fundamental mode antenna when the radiation efficiency is not too low.

When antennas generate multiple spherical modes, antennas generating higher-order spherical modes may be efficient for wireless power transfer. Because the magnitude of the imaginary part of $A_{\mu,nm}(r,\theta,\phi)$ is large when *n* or *v* is large, the magnitude of the imaginary part of *X* in (3) can be increased by using higher-order modes.

When antennas are not CMS antennas, a value is added to the Z-parameter calculated by (1) because of multiple reflections [15]. If the added value is not large, we can guess that the higher-order mode is also efficient when the antennas are not CMS antennas.

IV. Experiment

To verify the theory, we designed an antenna generating the TM_{01} mode and an antenna generating the TM_{02} mode. We then conducted a wireless power transfer experiment.

To increase the radiation efficiency of the antennas, we chose a folded cylindrical helix (FCH) [16], [17]. A four-arm 1/2 turn FCH antenna with a radius of 8 cm and a height of 21 cm was used as the TM₀₁ mode antenna (Fig. 4). A balun was connected to the feeding port of the antenna. The TM₀₂ mode antenna was composed of two four-arm 1/2 turn FCH antennas with a radius of 8 cm and a height of 8.5 cm. The axes of the two FCH antennas coincided, and the distance between the centers of the two antennas was 12.5 cm (Fig. 5). We excited each feeding port of the two element antennas so that the phase difference between the currents on the two antennas was 180°. To excite out of phase, the baluns connected to the feeding port were placed in opposite directions. The size of the TM₀₂ mode antenna was the same as that of the TM₀₁ mode antenna. All antennas were made of copper wire with a diameter of 1 mm.

We simulated the antennas with FEKO, which is based on the method of moments. The S-parameter of the feeding circuits were measured and used in the simulation. Table 1 shows the spherical mode coefficients generated by each of the antennas. When the spherical mode coefficients were computed, a reactance that matched the antenna was connected to it. In Table 1, the TM_{01} mode antenna generated mostly TM_{01} mode, and the TM_{02} mode antenna generated mostly TM_{01} mode. The radiation efficiency of the TM_{01} mode antenna was 81 % at 260 MHz, and the radiation efficiency of the TM_{02} mode antenna was 20 % at 274 MHz.



Fig. 4. TM₀₁ mode antenna.



(a) Element of the TM_{02} (b) Simulation model of the TM_{02} momode antenna de antenna and coordinate system



(c) Feeding circuit



(d) Fabricated antenna

Fig. 5. TM_{02} mode antenna.

We simulated and measured the S-parameter of two coupled TM_{01} mode antennas and the S-parameter of two coupled TM_{02} mode antennas. Here, θ and ϕ set to 0, and *r* was varied. Fig. 6 shows the simulated and measured S-parameter when the distance between the centers of the antennas is 50 cm. In the case of the TM_{02} mode antenna, the measured S-parameter changed slightly from the simulated S-parameter because of errors in the fabrication.

The maximum power transfer efficiency was calculated from the S-parameter using the simultaneous matching formula [18]. In the case of TM_{01} mode antenna, the maximum power transfer efficiency was the largest at 260 MHz. In the case of TM_{02} mode antenna, the maximum power transfer efficiency was the largest at 274 MHz in the simulation, while the maximum power transfer efficiency was the largest at 277 MHz in the

 Table 1. Spherical mode coefficients of the matched antennas.

(a) TM_{01} mode antenna at 260 MHz				
Mode	Coefficient			
TE_{-11}	$0.0234 ightarrow -10.2^\circ$			
TE_{11}	0.0235∠ −117.2°			
TM_{01}	0.898∠88.2°			
TE ₀₂	$0.0561 \angle -92.8^{\circ}$			

(0) (0) (0) (0) (0) (0) (0)	((b)	TM_{02}	mode	antenna	at	274	MH
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Mode	Coefficient
TE ₀₁	0.0380∠146.6°
TM_{02}	0.444∠147.1°
TE ₀₃	0.0332∠ -33.0°



Fig. 6. Simulated and measured S-parameters when the center-to-center distance between the antennas is 50 cm.



Fig. 7. Maximum power transfer efficiencies versus the center-to-center distance between the antennas obtained from the CMS theory, simulation, and measurement.

measurement. Fig. 7 shows the maximum power transfer efficiencies obtained from the CMS theory, simulation, and measurement. The curve for CMS was obtained using (3), (4), and the radiation efficiencies obtained with the simulation. The measured maximum power transfer efficiency was 0.26 at 260 MHz for the TM₀₁ mode antennas and 0.42 at 277 MHz for the TM₀₂ mode antennas when the center-to-center distance between the antennas was 30cm. The maximum power transfer efficiency for the TM₀₂ antenna increased by 62 % compared to the TM₀₁ mode antenna. Notice that the maximum power transfer efficiency for the TM₀₂ mode antenna is higher than that for the TM₀₁ mode antenna with a radiation efficiency of 1. The measurement agrees with the curve for CMS in the case of the TM₀₁ mode power transfer system, whereas there is an error between the measurement and the curve for CMS in the case of the TM₀₂ mode power transfer system, which was because the antennas are not CMS antennas.

Although the antennas used for the experiment were not CMS antennas, this experiment showed that the higher-order mode is more efficient than the fundamental mode.

V. Conclusion

To investigate the characteristics of wireless power transfer, we determined the Z-parameter between two CMS antennas. Using the formulas for the Z-parameter and the maximum power transfer efficiency, we showed that the power transfer efficiency increases as the mode number or radiation efficiency increases.

We experimented with the wireless power transfer using antennas generating TM_{01} mode and antennas gene-

rating TM_{02} mode. The maximum power transfer efficiency for two TM_{02} mode antennas was 62 % higher than that for two TM_{01} mode antennas when the center-to-center distance was 30 cm.

Although the theory was developed under the assumption that the antennas are CMS antennas, we demonstrated through the experiment that the higher-order mode antenna can be more efficient than the fundamental mode antenna even when the antennas are not CMS antennas.

It is important to note that the higher-order mode is not always efficient for wireless power transfer because the latter is influenced not only by the spherical modes, but also by the radiation efficiency. Therefore, in practice, the radiation efficiency of antennas should be considered when determining the best spherical mode to use.

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