Determination of the Impedance Parameters of Antennas and the Maximum Power Transfer Efficiency of Wireless Power Transfer

Yoon Goo Kim, *Member, IEEE*, and Sangwook Nam, *Senior Member, IEEE*

***Abstract*—In this paper, we derive formulas that calculate the impedance parameters (Z-parameters) of multiple antennas near objects to analyze wireless power transfer. Furthermore, we propose a method to calculate the impedance parameters of antennas, the maximum power transfer efficiency, and the optimum load impedance of a wireless power transfer system using equivalent current that generate the same electromagnetic field as that generated by an antenna in transmitting mode.**

***Index Terms*—Equivalent current, impedance parameters, maximum power transfer efficiency, wireless power transfer, Z-parameters.**

I. Introduction

RECENTLY, near-field wireless power transfer, that is, technologies that transfer electrical energy to devices using a near-field without being physically connected to an electric power source, has received considerable attention. To design an efficient near-field wireless power transfer system, we need to clearly understand its operating principles. One way to understand the operating principles of wireless power transfer is to develop and analyze the analytical models. Several analytical models have been proposed for near-field wireless power transfer. Two of the most frequently used analytical models are coupled mode theory [1] and equivalent circuit model [2], [3]. However, these models do not adequately explain the factors that affect the behavior of wireless power transfer. In other words, clearly identifying what properties of antennas determine the efficiency of wireless power transfer remains difficult. Furthermore, these models approximately analyze the wireless power transfer. Another analytical model for near-field wireless power transfer uses spherical waves [4], [5], which involves a complex calculation. In particular, in a case where large objects are present near the wireless power transfer system, the calculation using spherical waves is very complex and requires a longer time. Therefore, we need to develop a simple model that permits a thorough understanding of the operating principles of wireless power transfer. In this paper, we propose such a method to analyze wireless power transfer.

Manuscript received January 5, 2018; revised December 9, 2018; accepted January 25, 2019. Date of publication April 22, 2019; date of current version August 12, 2019. *(Corresponding author: Yoon Goo Kim.)*

Y. G. Kim was with the Department of Electrical and Computer Engineering, Seoul National University, Seoul 08826, South Korea. (e-mail: kygimoonoo@gmail.com).

S. Nam is with the Department of Electrical and Computer Engineering, Seoul National University, Seoul 08826, South Korea (e-mail: snam@snu.ac.kr).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TAP.2019.2912500

The relationship between the voltages and currents at the ports of antennas can be described by scattering parameters (S-parameters), impedance parameters (Z-parameters), or admittance parameters (Y-parameters). Once we know one of these parameters, we can determine the transferred power, the maximum power transfer efficiency, and the load impedance that maximizes the power transfer efficiency. Therefore, calculating the scattering, impedance, or admittance parameters for the ports of antennas is important for analyzing wireless power transfer. In this paper, we present formulas for calculating the impedance parameters of antennas based on induced electromotive force (EMF) method. Furthermore, we propose a method for calculating the formulas for impedance parameters using equivalent current that generate an electromagnetic field equivalent to that generated by an antenna.

One of the most important parameters in wireless power transfer is the power transfer efficiency, which depends on the load impedance connected to the receiving antenna. In this paper, we propose a method to calculate the maximum power transfer efficiency and load impedance that maximizes the power transfer efficiency.

In practice, objects such as floors, walls, tables, and chairs exist around the wireless power transfer system. Therefore, to model wireless power transfer in the real world, objects near the wireless power transfer system should be considered in the model. The method proposed in this paper can be applied to cases where objects are present near the wireless power transfer system.

Throughout this paper, it is assumed that electromagnetic fields are in a steady state. In addition, magnetic materials are assumed to be absent in antennas and objects. Furthermore, it is assumed that a source and load are connected across an infinitesimal gap on a conducting wire of an antenna, and that the antenna has one feed port. In this paper, the time convention $e^{j\omega t}$ is used, where ω is angular frequency and t is time.

II. Equivalent Circuit for an Antenna

Consider an antenna located near an objects. When an antenna is excited by a source at its feed port with no

0018-926X © 2019 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.
See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

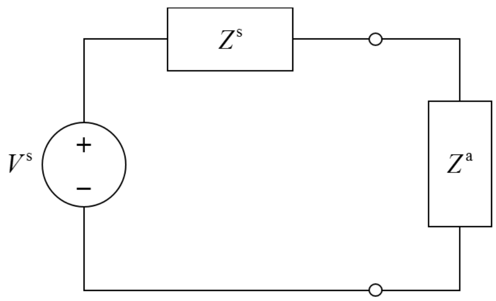


Fig. 1. Equivalent circuit for a transmitting antenna.

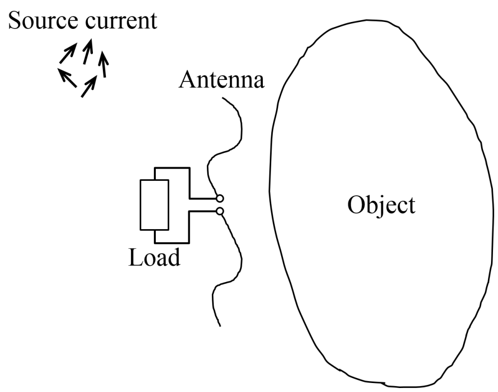


Fig. 2. Loaded antenna, object, and source current.

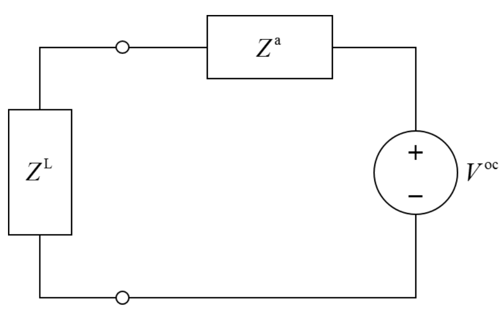


Fig. 3. Equivalent circuit for a receiving antenna.

incident electromagnetic field (antenna in transmitting mode), the voltage and current at the feed port can be calculated using an equivalent circuit, as shown in Fig. 1. In this figure, Z^{a} is the input impedance observed at the feed port of the antenna in the presence of objects, and Z^{s} and V^{s} are the source impedance and the open-circuit voltage of a Thevenin equivalent circuit for a source, respectively.

Consider that objects, source currents, and the antenna terminated in a load placed together and the source currents generating an electromagnetic field (Fig. 2). In this situation, the current flowing at a load and the voltage across that load can be calculated using a Thevenin equivalent circuit, as shown in Fig. 3. In this figure, Z^{a} is the input impedance observed at the feed port of an antenna in the presence of objects and in the absence of source currents, Z^{L} is the load impedance, and V^{oc} is the open-circuit voltage. The formula for calculating the open-circuit voltage V^{oc} can be derived from the reciprocity

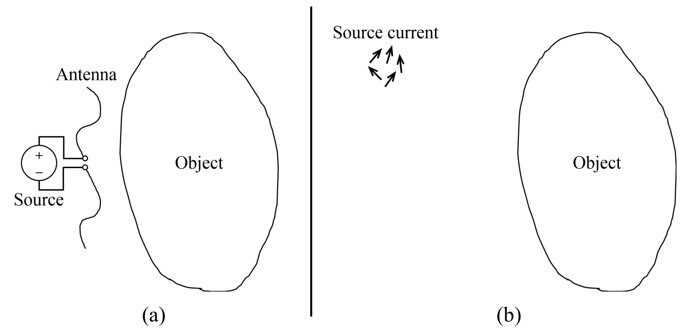


Fig. 4. Method for calculating the open-circuit voltage of equivalent circuit for a receiving antenna in the scenario in Fig. 2. (a) Situation in which the input impedance Z^{a} and electric current density $\mathbf{J}^{\mathrm{at}}$ of an antenna in transmitting mode is determined. (b) Situation in which the electric field $\mathbf{E}^{\mathrm{in}}$ is determined.

theorem [6, pp. 483–485]. The open-circuit voltage can be determined using the following formula:

$$V^{\mathrm{oc}} = -\frac{1}{I^{\mathrm{at}}}\int_R \mathbf{E}^{\mathrm{in}} \cdot \mathbf{J}^{\mathrm{at}} dv \tag{1}$$

where $\mathbf{J}^{\mathrm{at}}$ is the electric current density[1] of an antenna when its input terminals (feed port) are excited with a source in the presence of objects and in the absence of source currents; I^{at} is the total electric current flowing at the input terminal when the current density of the antenna is $\mathbf{J}^{\mathrm{at}}$; $\mathbf{E}^{\mathrm{in}}$ is the electric field that exists when objects and source currents are present and an antenna is absent; and R is a region containing the antenna (Fig. 4). In [6], the formula for the open-circuit voltage of an equivalent circuit for a receiving antenna was derived when the antenna is in free space and no objects are nearby. When objects exist near an antenna, (1) can be deduced from the reciprocity theorem.

While deriving the self-impedance and mutual impedance of antennas, an equivalent circuit for transmitting and receiving antennas will be exploited.

III. Input Impedance of an Antenna Near Objects

A. Calculation of the Input Impedance of an Antenna From the Current of a Transmitting Antenna

When an antenna is exclusively composed of a perfect electric conductor (PEC), the input impedance can be calculated using the induced EMF method. In this case, the input impedance of an antenna, Z^{a}, can be calculated using the following formula [6, p. 555], [7][2]:

$$Z^{\mathrm{a}} = -\frac{1}{(I^{\mathrm{at}})^2}\int_R \mathbf{E}^{\mathrm{at}} \cdot \mathbf{J}^{\mathrm{at}} dv \tag{2}$$

where $\mathbf{E}^{\mathrm{at}}$ is the electric field generated when an antenna near objects operates in transmitting mode, i.e., the electric field generated by $\mathbf{J}^{\mathrm{at}}$ and the currents of the objects.

[1] In a dielectric, the polarization current $j\omega(\varepsilon-\varepsilon_0)\mathbf{E}^{\mathrm{d}}$ is used as an electric current density; ε is the permittivity of the dielectric; ε_0 is the permittivity of free space; and $\mathbf{E}^{\mathrm{d}}$ is the electric field in the dielectric.

[2] In [6] and [7], the input impedance of an antenna was derived when the antenna is in free space. When objects are present near the antenna, (2) is obtained from the reciprocity theorem and the method in [6] or [7].

The integral in (2) can be written as

$$\int_R \mathbf{E}^{\mathrm{at}} \cdot \mathbf{J}^{\mathrm{at}} dv = \int_{R-gap} \mathbf{E}^{\mathrm{at}} \cdot \mathbf{J}^{\mathrm{at}} dv + \int_{gap} \mathbf{E}^{\mathrm{at}} \cdot \mathbf{J}^{\mathrm{at}} dv \quad (3)$$

where *gap* denotes a region that contains only a feed gap. Here, $\mathbf{J}^{\mathrm{at}}$ on the gap is defined as follows. A feed gap is filled with a PEC, and a circumferentially directed magnetic current is placed on the surface of the PEC that is filling the gap (delta gap model). In this case, the electric current density on the PEC filling gap is $\mathbf{J}^{\mathrm{at}}$ on the gap.

If an antenna is made of only PEC, then the tangential component of $\mathbf{E}^{\mathrm{at}}$ on the PEC is much smaller than the tangential component of $\mathbf{E}^{\mathrm{at}}$ on the feed gap, and the first integral on the right-hand side of (3) can, therefore, be neglected. The second integral on the right-hand side of (3) is approximated by $-V^{\mathrm{at}} I^{\mathrm{at}}$, where V^{at} is the voltage difference between the input terminals of an antenna, when the source is applied at the input terminals. Therefore, (2) is valid when an antenna is made of only a PEC.

If the loss of an antenna is large, then the first integral on the right-hand side of (3) is not negligible. Therefore, the input impedance of an antenna cannot be calculated using the previously induced EMF method when the loss of an antenna is large. The electric field and current density on a lossy conductor are related by

$$\mathbf{E}^{\tan} = Z^{\mathrm{sl}} \mathbf{J}^{\mathrm{sl}} \quad (4)$$

where $\mathbf{E}^{\tan}$ is the tangential component of the electric field to a surface (or line), $\mathbf{J}^{\mathrm{sl}}$ is the surface (or line) electric current density, and Z^{sl} is the surface (or line) impedance. When an antenna is made of only a lossy conductor, from (3) and (4), the input impedance of the antenna is as follows:

$$\begin{aligned} Z^{\mathrm{a}} &\approx -\frac{1}{(I^{\mathrm{at}})^2} \int_R \mathbf{E}^{\mathrm{at}} \cdot \mathbf{J}^{\mathrm{at}} dv + \frac{1}{(I^{\mathrm{at}})^2} \int_{R-gap} Z^{\mathrm{sl}} \mathbf{J}^{\mathrm{at}} \cdot \mathbf{J}^{\mathrm{at}} dv \\ &\approx -\frac{1}{(I^{\mathrm{at}})^2} \int_R (\mathbf{E}^{\mathrm{at}} - Z^{\mathrm{sl}} \mathbf{J}^{\mathrm{at}}) \cdot \mathbf{J}^{\mathrm{at}} dv. \end{aligned} \quad (5)$$

For the second integral in (5), integrations over the R-gap and R are almost the same because the gap is very small. Equation (5) is used to calculate the input impedance of an antenna when the current distribution of an antenna in transmitting mode is known.

B. Variation of the Input Impedance of an Antenna Due to Objects

In this paper, objects are classified into two types: type A and type B. Type A object exists with an antenna when determining the current distribution of an antenna in transmitting mode ($\mathbf{J}^{\mathrm{at}}$), which is used for the calculation of the open-circuit voltage of an equivalent circuit for a receiving antenna (V^{oc}). Type B objects do not exist when the current distribution of an antenna in transmitting mode is determined to calculate the open-circuit voltage of an equivalent circuit for a receiving antenna. Therefore, when the current distribution of an antenna in transmitting mode ($\mathbf{J}^{\mathrm{at}}$) is determined, the antenna is excited by a source at a feed port in the presence of a type A object and in the absence of type B objects; then the current of the antenna is calculated. Thus, the object referred to in Section II is a type A object. A type A object is always present when the current distributions of antennas is determined to calculate their impedance parameters. For example, an environment such as ground is set as a type A object, and obstacles around the antennas are set as type B objects.

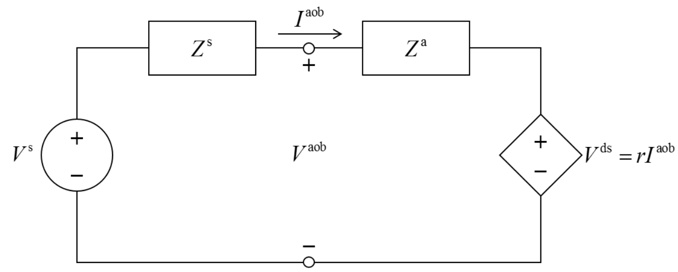


Fig. 5. Equivalent circuit for a transmitting antenna with type A and type B objects.

We will derive the difference between the input impedance of an antenna in the presence of a type A object and absence of type B objects (Z^{a}), and in the presence of both type A and type B objects (Z^{aob}). In the case where an antenna and a type A object exist and type B objects do not, an antenna in transmitting mode can be modeled using an equivalent circuit for a transmitting antenna (Fig. 1). In the case where an antenna and both type A and type B objects exist, and the antenna is excited with a source at its feed port, the electromagnetic field transmitted from an antenna is scattered by the type B objects and this scattered field is incident on the antenna. In this case, the voltage due to the scattered field, as well as the voltage due to the source, is produced at the feed port of the antenna. Antennas with type A and B objects can be modeled such that the current-controlled voltage source is connected to Z^{a} in the equivalent circuit for a transmitting antenna, as shown in Fig. 5. The voltage generated by the dependent voltage source is proportional to the current flowing at the feed port because the strength of the scattered field is proportional to the current at the feed port.

The voltage generated by the dependent voltage source (V^{ds}) can be calculated using the method for calculating the open-circuit voltage (V^{oc}) in the Thevenin equivalent circuit for a receiving antenna. The currents of type B objects play the role of the source currents in Section II. The voltage generated by the dependent source is calculated as follows. First, when an antenna is present along with a type A and type B objects, it is excited at a feed port with a source. In this situation, the currents of the type B objects in steady state are determined. Next, the antenna and type B objects are eliminated but the type A object and currents of the type B objects remain. The electric field is calculated in the situation where the type A object and the currents of the type B objects are present. Next, the voltage V^{ds} is calculated using the electric field and (1).

The input impedance of an antenna is the ratio of the voltage to the current at the feed port. Let the voltage and current at the feed port, when an antenna is excited at its feed port in the presence of type A and type B objects, be V^{aob} and I^{aob}, respectively. Solving the circuit in Fig. 5, the port current $I^{\rm aob}$ is

$$I^{\rm aob} = \frac{V^{\rm aob} - V^{\rm ds}}{Z^{\rm a}}. \tag{6}$$

Therefore, the input impedance of an antenna in the presence of both type A and type B objects, $Z^{\rm aob}$, is

$$Z^{\rm aob} = \frac{V^{\rm aob}}{I^{\rm aob}} = \frac{V^{\rm aob} Z^{\rm a}}{V^{\rm aob} - V^{\rm ds}}. \tag{7}$$

Calculating $Z^{\rm aob} - Z^{\rm a}$

$$Z^{\rm aob} - Z^{\rm a} = \frac{V^{\rm aob} Z^{\rm a}}{V^{\rm aob} - V^{\rm ds}} - Z^{\rm a} = \frac{V^{\rm ds} Z^{\rm a}}{V^{\rm aob} - V^{\rm ds}} = \frac{V^{\rm ds}}{I^{\rm aob}}. \tag{8}$$

From (1) and (8), a formula for the difference between the input impedance of an antenna in the presence of a type A object but no type B object and in the presence of both type A and type B objects can be given as follows:

$$Z^{\rm aob} - Z^{\rm a} = -\frac{1}{I^{\rm aob} I^{\rm at}} \int_R \mathbf{E}^{\rm ob} \cdot \mathbf{J}^{\rm at} dv \tag{9}$$

where $\mathbf{E}^{\rm ob}$ is the electric field generated from the currents of type B objects and a type A object when an antenna is excited at its input terminals in a situation where all, antenna, type A, and type B objects, are present. The electric field generated by the current of the antenna is not included in $\mathbf{E}^{\rm ob}$. $\mathbf{J}^{\rm at}$ is the electric current density of an antenna when its input terminals are excited in the presence of a type A object and absence of type B objects. $I^{\rm at}$ is the total electric current at the input terminal when the current density of an antenna is $\mathbf{J}^{\rm at}$. Note that $\mathbf{E}^{\rm ob}$ is proportional to $I^{\rm aob}$ and $\mathbf{J}^{\rm at}$ is proportional to $I^{\rm at}$. $\mathbf{E}^{\rm ob}$ can be considered as an electric field generated by type A and type B objects when an ideal current source generating $I^{\rm aob}$ is connected to the feed port of an antenna.

To calculate the input impedance of an antenna with both type A and type B objects ($Z^{\rm aob}$) using (9), the electric field scattered by the type B objects ($\mathbf{E}^{\rm ob}$) is required. If we calculate the electric field $\mathbf{E}^{\rm ob}$ in a situation where a transmitting antenna, type A, and type B objects exist, then we can easily find the input impedance of an antenna with both type A and B objects, $Z^{\rm aob}$. Therefore, in this case, (9) is not required to calculate $Z^{\rm aob}$. In some cases, we can calculate $\mathbf{E}^{\rm ob}$ without determining the electric field in the presence of a transmitting antenna and both type A and type B objects. If the antenna and type B objects are not electrically very close, then the shape of the current distribution of a transmitting antenna with both type A and type B objects is similar to that of a transmitting antenna with only a type A object. In this case, $\mathbf{E}^{\rm ob}$ can be approximately calculated using the current of a transmitting antenna with only a type A object ($\mathbf{J}^{\rm at}$) rather than the current of a transmitting antenna with both type A and type B objects.

Therefore, the input impedance of an antenna with both type A and type B objects can be calculated using the current distribution of a transmitting antenna with a type A object and without type B objects, if the antenna and type B objects are electrically not very close.

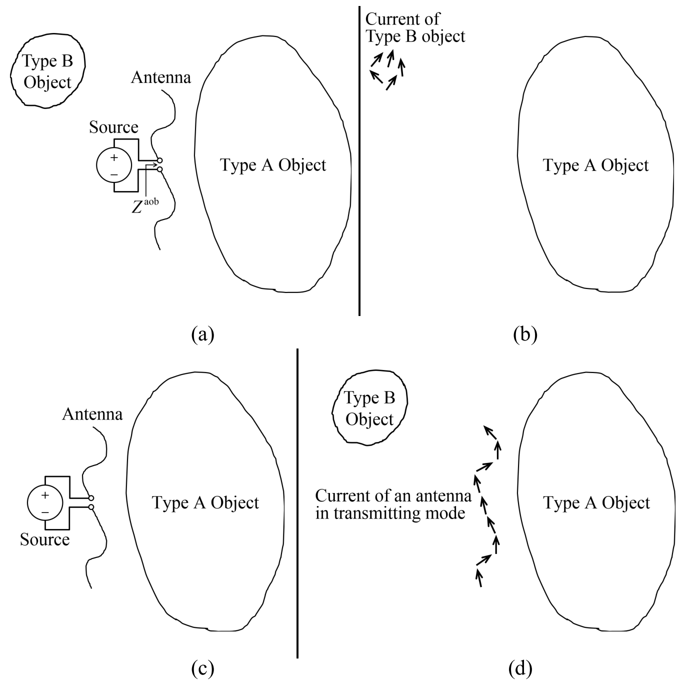


Fig. 6. Method for calculating the input impedance of an antenna near objects. (a) Situation in which the impedance $Z^{\rm aob}$ is determined. (b) Situation in which the electric field $\mathbf{E}^{\rm ob}$ is determined. The current density of type B objects is determined in the situation in (a). (c) Situation in which the input impedance $Z^{\rm a}$ and electric current density $\mathbf{J}^{\rm at}$ of an antenna in transmitting mode is determined. (d) Situation in which the current density of type B objects is approximately determined when an antenna and type B objects are not very close electrically. The current of an antenna in transmitting mode are determined in the situation in (c).

IV. Self and Mutual Impedance of Coupled Antennas

A. Mutual Impedance Between Antennas

The relationship between the voltages and currents at ports can be described using the impedance parameters as follows:

$$V_m = \sum_{n=1}^{N} Z_{mn} I_n \tag{10}$$

where V_m is the voltage at the port of the mth antenna, I_n is the current at the port of the nth antenna, Z_{mn} is the impedance parameter (or Z-parameter), and N is the total number of ports. In this paper, the nth antenna is denoted as antenna n. The impedance parameters can be determined as

$$Z_{mn} = \frac{V_m}{I_n} \quad \text{when } I_i = 0 \quad \text{for } i \neq n. \tag{11}$$

When $m \neq n$, Z_{mn} is called a mutual impedance. In (11), V_m is equivalent to the voltage at the feed port of antenna m when antenna n is excited at its port by the current of I_n, and all other antennas are open-circuited. When $m \neq n$, V_m in (11) can be obtained by using the method for calculating the open-circuit voltage in the Thevenin equivalent circuit for an antenna in receiving mode. The currents of the type B objects and antennas, except for antenna m, play the role of the source currents in Section II. From (1) and (11), the mutual impedance, Z_{mn}, is as follows:

$$Z_{mn} = -\frac{1}{I_n I_m^{\rm at}} \int_{R_m} \mathbf{E}_{mn} \cdot \mathbf{J}_m^{\rm at} dv \tag{12}$$

where

$$\mathbf{E}_{mn} = \mathbf{G}^{\rm oa}\left(\mathbf{J}_{{\rm a},n}^{{\rm ex},n}\right) + \left[\sum_{i \neq n,m} \mathbf{G}^{\rm oa}\left(\mathbf{J}_{{\rm ao},i}^{{\rm ex},n}\right)\right] + \mathbf{G}^{\rm oa}\left(\mathbf{J}_{\rm ob}^{{\rm ex},n}\right) \tag{13}$$

for $m \neq n$. In (12), $\mathbf{J}_m^{\rm at}$ is the electric current density of antenna m when antenna m is excited at its input terminals with a source, in the presence of a type A object and in the absence of other antennas and type B objects; $I_m^{\rm at}$ is the total electric current flowing at the input terminal of antenna m when the current density of antenna m is $\mathbf{J}_m^{\rm at}$; and R_m is a region that contains only antenna m. $\mathbf{E}_{mn}$ is the electric field at antenna m due to the currents of the other antennas and type A and type B objects when antenna n is excited at its input terminals and antennas except n are open-circuited. $\mathbf{E}_{mn}$ is calculated as follows: first, in a situation where all antennas and type A and type B objects exist, antennas other than antenna n are open-circuited, and antenna n is excited at the input terminals. Then, the current distributions of the antennas and the type B objects in the steady state are calculated. Second, the antennas and type B objects are eliminated, but the currents of the antennas, except antenna m, and the currents of the type B objects remain (the current of antenna m is eliminated). The electric field is calculated from the currents of the antennas except antenna m and the currents of the type B objects in the presence of a type A object. I_n is the value of the total electric current at the input terminal of antenna n with open-circuited antennas and type A and B objects when the electric field is $\mathbf{E}_{mn}$. In (13), $\mathbf{G}^{\rm oa}$ is the dyadic Green's function when only a type A object exists. $\mathbf{J}_{{\rm a},n}^{{\rm ex},n}$, $\mathbf{J}_{{\rm ao},i}^{{\rm ex},n}$, and $\mathbf{J}_{\rm ob}^{{\rm ex},n}$ are the electric current densities of antenna n, open-circuited antenna i, and the type B objects, respectively, when antenna n is excited at its input terminals so that the current at the input terminal is I_n in a situation where the antennas, type A object, and type B objects are all present, and the antennas, except n are open-circuited. Note that $\mathbf{E}_{mn}$ is proportional to I_n and $\mathbf{J}_m^{\rm at}$ is proportional to $I_m^{\rm at}$.

Using a reciprocity theorem [8, eq. (3–36)], (12) can be written in another form

$$Z_{mn} = -\frac{1}{I_n I_m^{\rm at}} \int_{R_{mn}} \mathbf{E}_m^{\rm at} \cdot \mathbf{J}_{mn} dv \tag{14}$$

where $\mathbf{E}_m^{\rm at}$ is the electric field generated when antenna m is excited at the input terminals so that the current at the input terminal is $I_m^{\rm at}$ in a situation in which only antenna m and a type A object exist, i.e., $\mathbf{E}_m^{\rm at}$ is generated by $\mathbf{J}_m^{\rm at}$ and a type A object. $\mathbf{J}_{mn}$ is the electric current density of the type B objects and antennas, except for antenna m, when the antennas except antenna n are open-circuited and antenna n is excited by an ideal current source generating I_n, i.e., $\mathbf{J}_{mn} = \mathbf{J}_{{\rm a},n}^{{\rm ex},n} + [\sum_{i \neq n,m} \mathbf{J}_{{\rm ao},i}^{{\rm ex},n}] + \mathbf{J}_{\rm ob}^{{\rm ex},n}$ $(m \neq n)$. That is, when $\mathbf{J}_{mn}$ and a type A object exist, the electric field $\mathbf{E}_{mn}$ is generated. R_{mn} is a region containing $\mathbf{J}_{mn}$.

To calculate the mutual impedances using the method presented in this paper, we must know the current distributions of all antennas. In some cases, we can predict the current distributions of antennas without calculating the currents in the situation where all antennas and all objects are present. When antennas and type B objects are electrically not very close and antennas are electrically small, the shape of the current distribution of the energized antenna, in the presence of open-circuited antennas and both type A and type B objects ($\mathbf{J}_{{\rm a},n}^{{\rm ex},n}$), is similar to the shape of the current distribution of an antenna that is energized in the absence of other antennas and type B objects and in the presence of a type A object ($\mathbf{J}_n^{\rm at}$). If an antenna is small compared to the wavelength, then the strength of the electromagnetic field scattered by an open-circuited antenna is much smaller than the strength of the electromagnetic field generated by an energized antenna. Thus, if the antennas and type B objects are electrically not very close, open-circuited small antennas can be neglected when

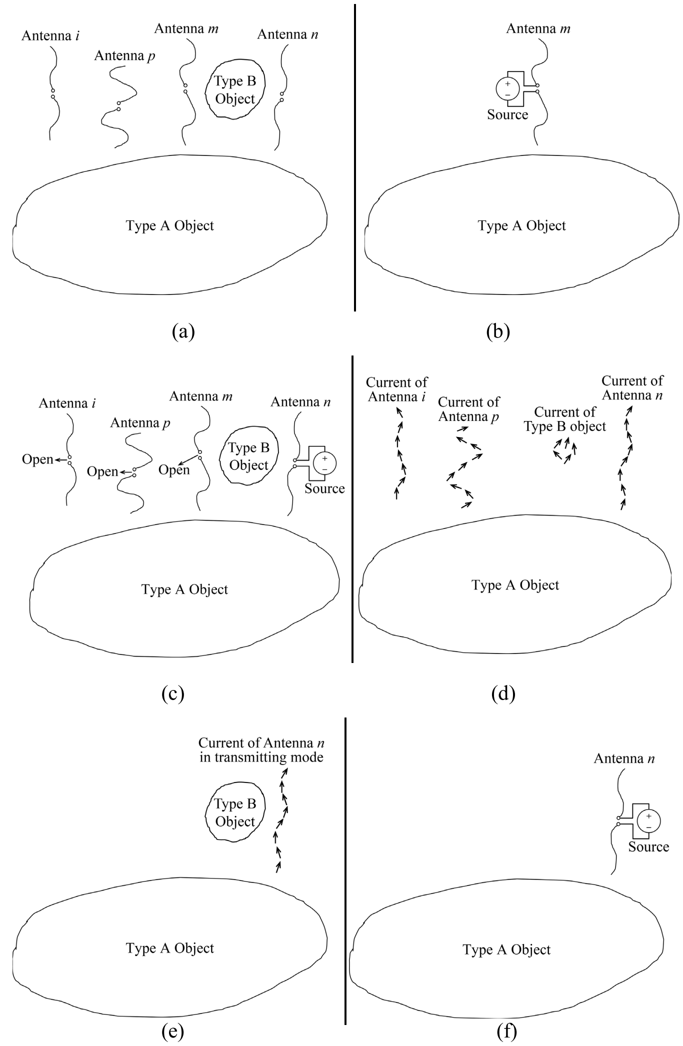


Fig. 7. Method for calculating the mutual impedance Z_{mn}. (a) Situation in which impedance parameters of antennas is determined. (b) Situation in which $\mathbf{J}_m^{\rm at}$ is determined. (c) Situation in which $\mathbf{J}_{{\rm a},n}^{{\rm ex},n}$, $\mathbf{J}_{{\rm ao},i}^{{\rm ex},n}$, and $\mathbf{J}_{\rm ob}^{{\rm ex},n}$ is determined. (d) Situation in which $\mathbf{E}_{mn}$ is determined. Currents of antennas and type B objects are determined in situation in (c). (e) Situation in which $\mathbf{E}_{mn}$ is approximately determined when antennas and a type B object are not very close electrically and antennas are electrically small. (f) Situation in which the current of the antenna in transmitting mode in (e) is determined.

calculating $\mathbf{E}_{mn}$. Therefore, when the antennas are smaller than the wavelength and the antennas and type B objects are electrically not very close, $\mathbf{E}_{mn}$ can be approximated by

$$\frac{\mathbf{E}_{mn}}{I_n} \approx \frac{\mathbf{G}^{\mathrm{oa}}(\mathbf{J}_n^{\mathrm{at}}) + \mathbf{G}^{\mathrm{oa}}(\mathbf{J}_{\mathrm{ob}}^{\mathrm{at},n})}{I_n^{\mathrm{at}}} \tag{15}$$

where $\mathbf{J}_{\mathrm{ob}}^{\mathrm{at},n}$ is the current density of the type B objects that is induced when only type A, type B objects, and $\mathbf{J}_n^{\mathrm{at}}$ exist. If no type B objects exist or if the field due to the type B objects is much smaller than the field due to the energized antenna, then

$$\frac{\mathbf{E}_{mn}}{I_n} \approx \frac{\mathbf{E}_n^{\mathrm{at}}}{I_n^{\mathrm{at}}}. \tag{16}$$

Therefore, in the case where the antennas are small compared to the wavelength and the antennas and type B objects are electrically not very close, the mutual impedance can be calculated using only the current distributions of antennas in transmitting mode in the absence of other antennas and type B objects and in the presence of a type A object.

B. Self-Impedance of Antennas

In this paper, Z_{mn} is called the self-impedance when $m = n$. The self-impedance Z_{mm} is equivalent to the impedance observed at the feed port of antenna m when all antennas and both type A and type B objects exist and all antennas, except antenna m are open-circuited. The self-impedance can be calculated using the same method used to calculate the input impedance of an antenna near objects. When calculating the self-impedance, the open-circuited antennas are considered type B objects. From (9), the self-impedance Z_{mm} is as follows:

$$Z_{mm} = Z_m^{\mathrm{a}} - \frac{1}{I_m I_m^{\mathrm{at}}} \int_{R_m} \mathbf{E}_{mm} \cdot \mathbf{J}_m^{\mathrm{at}} dv \tag{17}$$

where Z_m^{a} is the input impedance observed at the feed port of antenna m when type A objects exist with no other antennas and type B objects; and

$$\mathbf{E}_{mm} = \left[\sum_{i \neq m} \mathbf{G}^{\mathrm{oa}}(\mathbf{J}_{\mathrm{ao},i}^{\mathrm{ex},m})\right] + \mathbf{G}^{\mathrm{oa}}(\mathbf{J}_{\mathrm{ob}}^{\mathrm{ex},m}). \tag{18}$$

Using the reciprocity theorem [8, eq. (3–36)], (17) can be written in another form

$$Z_{mm} = Z_m^{\mathrm{a}} - \frac{1}{I_m I_m^{\mathrm{at}}} \int_{R_{mm}} \mathbf{E}_m^{\mathrm{at}} \cdot \mathbf{J}_{mm} dv \tag{19}$$

where $\mathbf{J}_{mm} = [\sum_{i \neq m} \mathbf{J}_{\mathrm{ao},i}^{\mathrm{ex},m}] + \mathbf{J}_{\mathrm{ob}}^{\mathrm{ex},m}$.

V. Calculation of the Impedance Parameters of Antennas Using Equivalent Currents

Let $\mathbf{J}_m^{\mathrm{eq}}$ be the electric current density that generates the same electromagnetic field as that generated by $\mathbf{J}_m^{\mathrm{at}}$. Let $\mathbf{E}_m^{\mathrm{eq}}$ be the electric field generated when $\mathbf{J}_m^{\mathrm{eq}}$ and a type A object exist, i.e., the value of $\mathbf{E}_m^{\mathrm{eq}}$ is equivalent to that of $\mathbf{E}_m^{\mathrm{at}}$. Let $\mathbf{J}_{mn}^{\mathrm{eq}}$ be the electric current density that generates the same electromagnetic field as that generated by $\mathbf{J}_{mn}$. Let $\mathbf{E}_{mn}^{\mathrm{eq}}$ be the electric field generated when $\mathbf{J}_{mn}^{\mathrm{eq}}$ and a type A object exist, i.e., the value of $\mathbf{E}_{mn}^{\mathrm{eq}}$ is equivalent to that of $\mathbf{E}_{mn}$. We shall call $\mathbf{J}_m^{\mathrm{eq}}$ and $\mathbf{J}_{mn}^{\mathrm{eq}}$ equivalent current. Replacing $\mathbf{E}_m^{\mathrm{at}}$ in (14) by $\mathbf{E}_m^{\mathrm{eq}}$ and applying the reciprocity theorem to (14) leads to

$$Z_{mn} = -\frac{1}{I_n I_m^{\mathrm{at}}} \int_{R_m^{\mathrm{eq}}} \mathbf{E}_{mn} \cdot \mathbf{J}_m^{\mathrm{eq}} dv. \tag{20}$$

Here, R_m^{eq} is a region containing $\mathbf{J}_m^{\mathrm{eq}}$. Next, replace $\mathbf{E}_{mn}$ in (20) with $\mathbf{E}_{mn}^{\mathrm{eq}}$; and the following equation is obtained:

$$Z_{mn} = -\frac{1}{I_n I_m^{\mathrm{at}}} \int_{R_m^{\mathrm{eq}}} \mathbf{E}_{mn}^{\mathrm{eq}} \cdot \mathbf{J}_m^{\mathrm{eq}} dv \tag{21}$$

for $m \neq n$. Likewise, using the reciprocity theorem and equivalent currents, (19) becomes as follows:

$$Z_{mm} = Z_m^{\mathrm{a}} - \frac{1}{I_m I_m^{\mathrm{at}}} \int_{R_m^{\mathrm{eq}}} \mathbf{E}_{mm}^{\mathrm{eq}} \cdot \mathbf{J}_m^{\mathrm{eq}} dv. \tag{22}$$

Equivalent current can be composed of infinitesimal electric dipoles or electric loops. Let I^{e} be the value of the current of an electric dipole and l be the length of an electric dipole. Let $\mathbf{e}$ be a vector whose direction is equal to the direction of the flow of current in an electric dipole. Let the magnitude of $\mathbf{e}$ be $|I^{\mathrm{e}}l|$, and let the phase of each component of $\mathbf{e}$ be equal to the phase of I^{e}. In this paper, we shall call $I^{\mathrm{e}}l$ the electric moment and $\mathbf{e}$ the electric moment vector. Let I^{h} be the value of the current of an electric loop and A be the area of the circle bounded by the current of an electric loop. Let $\mathbf{m}$ be a vector whose direction is perpendicular to the plane in which the loop lies. Let the magnitude of $\mathbf{m}$ be $|I^{\mathrm{h}}A|$, and let the phase of each component of $\mathbf{m}$ be equal to the phase of I^{h}. In this paper, we shall call $I^{\mathrm{h}}A$ the magnetic moment and $\mathbf{m}$ the magnetic moment vector.

Suppose $\mathbf{J}_m^{\mathrm{eq}}$ in (21) and (22) is composed of one infinitesimal electric dipole with an electric moment vector $\mathbf{e}$ that is located at $\mathbf{x}$, where $\mathbf{x}$ is a position vector. Calculating the integral in (21) and (22), we obtain

$$\int_{R_m^{\mathrm{eq}}} \mathbf{E}_{mn}^{\mathrm{eq}} \cdot \mathbf{J}_m^{\mathrm{eq}} dv \approx \mathbf{E}_{mn}^{\mathrm{eq}}(\mathbf{x}) \cdot \mathbf{e} \tag{23}$$

where n can either be equal to or different from m. Suppose $\mathbf{J}_m^{\mathrm{eq}}$ in (21) and (22) is composed of one infinitesimal electric loop with a magnetic moment vector $\mathbf{m}$ that is located at $\mathbf{y}$, where $\mathbf{y}$ is a position vector. Calculating the integral in (21) and (22), we obtain

$$\begin{aligned}\int_{R_m} \mathbf{E}_{mn}^{\mathrm{eq}} \cdot \mathbf{J}_m^{\mathrm{eq}} dv &= I^h \oint_{\partial C} \mathbf{E}_{mn}^{\mathrm{eq}} \cdot d\mathbf{l} = -j\omega\mu I^h \iint_C \mathbf{H}_{mn}^{\mathrm{eq}} \cdot d\mathbf{s} \\ &\approx -j\omega\mu \mathbf{H}_{mn}^{\mathrm{eq}}(\mathbf{y}) \cdot \mathbf{m}\end{aligned} \tag{24}$$

where $\mathbf{H}_{mn}^{\mathrm{eq}}$ is the magnetic field when the electric field is $\mathbf{E}_{mn}^{\mathrm{eq}}$ and C is the region bounded by a circle at which the current flows.

If $\mathbf{J}_m^{\mathrm{eq}}$ is composed of multiple infinitesimal electric dipoles and electric loops, then the values of the integrals in (21) and (22) are obtained by the superposition of results for all dipoles and loops. Therefore, the result of the integral in the case of multiple infinitesimal dipoles and loops is as follows:

$$\int_{R_m^{\mathrm{eq}}} \mathbf{E}_{mn}^{\mathrm{eq}} \cdot \mathbf{J}_m^{\mathrm{eq}} dv = \sum_{i=1}^{N_{\mathrm{e}}^{\mathrm{a},m}} \left(\mathbf{E}_{mn}^{\mathrm{eq}}\left(\mathbf{x}_i^{\mathrm{a},m}\right) \cdot \mathbf{e}_i^{\mathrm{a},m}\right) - j\omega\mu \sum_{i=1}^{N_{\mathrm{h}}^{\mathrm{a},m}} \left(\mathbf{H}_{mn}^{\mathrm{eq}}\left(\mathbf{y}_i^{\mathrm{a},m}\right) \cdot \mathbf{m}_i^{\mathrm{a},m}\right) \quad (25)$$

where $\mathbf{e}_i^{\mathrm{a},m}$ is the electric moment vector of the ith infinitesimal dipole for antenna m when its port current is I_m^{at}, and $\mathbf{m}_i^{\mathrm{a},m}$ is the magnetic moment vector of the ith infinitesimal loop for antenna m when its port current is I_m^{at}; $\mathbf{x}_i^{\mathrm{a},m}$ is the position of the ith infinitesimal dipole for antenna m, and $\mathbf{y}_i^{\mathrm{a},m}$ is the position of the ith infinitesimal loop for antenna m; $N_{\mathrm{e}}^{\mathrm{a},m}$ is the total number of infinitesimal dipoles for antenna m, and $N_{\mathrm{h}}^{\mathrm{a},m}$ is the total number of infinitesimal loops for antenna m. A collection of all $\mathbf{e}_i^{\mathrm{a},m}$ and all $\mathbf{m}_i^{\mathrm{a},m}$ (m is fixed to one value) generates the same field as that generated by antenna m in transmitting mode with a port current of I_m^{at}. Note that $\mathbf{e}_i^{\mathrm{a},m}$ and $\mathbf{m}_i^{\mathrm{a},m}$ are proportional to I_m^{at}.

Let $\mathbf{E}_{\mathrm{eob}}^{\mathrm{eq},n}$ and $\mathbf{H}_{\mathrm{eob}}^{\mathrm{eq},n}$ be the electric and magnetic fields, respectively, generated when $\mathbf{J}_n^{\mathrm{eq}}$, a type A object, and type B objects exist but no other currents does; that is, $\mathbf{E}_{\mathrm{eob}}^{\mathrm{eq},n} = \mathbf{G}^{\mathrm{oa}}(\mathbf{J}_n^{\mathrm{eq}}) + \mathbf{G}^{\mathrm{oa}}(\mathbf{J}_{\mathrm{ob}}^{\mathrm{eq},n})$, where $\mathbf{J}_{\mathrm{ob}}^{\mathrm{eq},n}$ is the current density of the type B objects induced when a type A object, type B objects, and $\mathbf{J}_n^{\mathrm{eq}}$ exist. Let $\mathbf{E}_{\mathrm{ob}}^{\mathrm{eq},n}$ and $\mathbf{H}_{\mathrm{ob}}^{\mathrm{eq},n}$ be the electric field and magnetic fields, respectively, generated when there are a type A object and currents of type B objects induced in the situation where $\mathbf{J}_n^{\mathrm{eq}}$, a type A object, and type B objects exist; that is, $\mathbf{E}_{\mathrm{ob}}^{\mathrm{eq},n} = \mathbf{G}^{\mathrm{oa}}(\mathbf{J}_{\mathrm{ob}}^{\mathrm{eq},n})$. The electric field generated from $\mathbf{J}_n^{\mathrm{eq}}$ is not included in $\mathbf{E}_{\mathrm{ob}}^{\mathrm{eq},n}$. If the antennas are electrically small and are not close to the type B objects such that the influence of antennas and the type B objects on the shape of the current distribution of the antenna can be neglected, then $\mathbf{E}_{mn}^{\mathrm{eq}}/I_n$ and $\mathbf{H}_{mn}^{\mathrm{eq}}/I_n$ can be approximated by $\mathbf{E}_{\mathrm{eob}}^{\mathrm{eq},n}/I_n^{\mathrm{at}}$ and $\mathbf{H}_{\mathrm{eob}}^{\mathrm{eq},n}/I_n^{\mathrm{at}}$, respectively, when $m \neq n$, and $\mathbf{E}_{nn}^{\mathrm{eq}}/I_n$ and $\mathbf{H}_{nn}^{\mathrm{eq}}/I_n$ can be approximated by $\mathbf{E}_{\mathrm{ob}}^{\mathrm{eq},n}/I_n^{\mathrm{at}}$ and $\mathbf{H}_{\mathrm{ob}}^{\mathrm{eq},n}/I_n^{\mathrm{at}}$, respectively, because open-circuited electrically small antennas are invisible, that is,

$$\frac{\mathbf{E}_{mn}^{\mathrm{eq}}}{I_n} \approx \frac{\mathbf{E}_{\mathrm{eob}}^{\mathrm{eq},n}}{I_n^{\mathrm{at}}}, \quad \frac{\mathbf{H}_{mn}^{\mathrm{eq}}}{I_n} \approx \frac{\mathbf{H}_{\mathrm{eob}}^{\mathrm{eq},n}}{I_n^{\mathrm{at}}}$$
$$\frac{\mathbf{E}_{nn}^{\mathrm{eq}}}{I_n} \approx \frac{\mathbf{E}_{\mathrm{ob}}^{\mathrm{eq},n}}{I_n^{\mathrm{at}}}, \quad \frac{\mathbf{H}_{nn}^{\mathrm{eq}}}{I_n} \approx \frac{\mathbf{H}_{\mathrm{ob}}^{\mathrm{eq},n}}{I_n^{\mathrm{at}}} \quad (m \neq n). \quad (26)$$

If no type B object exists or if the influence of the type B object on the field is small, then $\mathbf{E}_{mn}^{\mathrm{eq}}/I_n \approx \mathbf{E}_n^{\mathrm{eq}}/I_n^{\mathrm{at}}$, $\mathbf{H}_{mn}^{\mathrm{eq}}/I_n \approx \mathbf{H}_n^{\mathrm{eq}}/I_n^{\mathrm{at}}$, $\mathbf{E}_{nn}^{\mathrm{eq}} \approx 0$, and $\mathbf{H}_{nn}^{\mathrm{eq}} \approx 0$, where $\mathbf{H}_n^{\mathrm{eq}}$ is magnetic field when the electric field is $\mathbf{E}_n^{\mathrm{eq}}$.

VI. Maximum Power Transfer Efficiency and Optimum Load Impedance

A. Maximum Power Transfer Efficiency

In this section, we will derive a formula that calculates the maximum power transfer efficiency between two antennas using equivalent currents. In this paper, the power transfer efficiency is defined as the power dissipated at the load in the receiving antenna divided by the power accepted by the transmitting antenna. A load is connected to a receiving antenna, and a source is connected to a transmitting antenna. If the impedance parameters of the two antennas are known, the maximum power transfer efficiency can be calculated. The formula for calculating the maximum power transfer efficiency ($PTE^{\max}$) is as follows [5]:

$$PTE^{\max} = \frac{|X_2|^2}{2-\mathrm{Re}(X_1X_2) + \sqrt{4-4\mathrm{Re}(X_1X_2)-\mathrm{Im}(X_1X_2)^2}} \quad (27)$$

where

$$X_1 = \frac{Z_{12}}{\sqrt{\mathrm{Re}(Z_{11})\mathrm{Re}(Z_{22})}}, \quad X_2 = \frac{Z_{21}}{\sqrt{\mathrm{Re}(Z_{11})\mathrm{Re}(Z_{22})}}. \quad (28)$$

If the antennas and objects are reciprocal, then the maximum power transfer efficiency is

$$PTE^{\max} = \frac{|X|^2}{2-\mathrm{Re}(X^2) + \sqrt{4-4\mathrm{Re}(X^2)-\mathrm{Im}(X^2)^2}} \quad (29)$$

where

$$X = \frac{Z_{21}}{\sqrt{\mathrm{Re}(Z_{11})\mathrm{Re}(Z_{22})}} = \frac{Z_{12}}{\sqrt{\mathrm{Re}(Z_{11})\mathrm{Re}(Z_{22})}}. \quad (30)$$

Substituting (21) and (22) into (30), we obtain (31), as shown at the top of the next page.

We assume that the antennas are small compared with the wavelength and the antennas and a type B object are not very close to each other so that they do not affect the shape of the current distribution of an antenna, i.e., (26) is assumed.

The values of the port currents I_1, I_2, I_1^{at}, and I_2^{at} are arbitrary. We set I_1^{at} and I_2^{at} to be equal to I_1 and I_2, respectively. Multiplying the numerator and denominator in (31) by $|I_1^{\mathrm{at}}||I_2^{\mathrm{at}}|$ and using (26), we obtain (32), as shown at the top of the next page, where $\arg(I_1^{\mathrm{at}})$ and $\arg(I_2^{\mathrm{at}})$ are the phases of I_1^{at} and I_2^{at}, respectively; P_1^{rad} is the power radiated when antenna 1 operates in transmitting mode and the port current of antenna 1 is I_1^{at} in the situation where only antenna 1 and a type A object are present; P_2^{rad} is the power radiated when antenna 2 operates in transmitting mode and the port current of antenna 2 is I_2^{at} in the situation where only antenna 2 and a type A object are present; η_1^{rad} is the radiation efficiency of antenna 1 with a type A object, and η_2^{rad} is the radiation efficiency of antenna 2 with a type A object. When the radiation efficiency is calculated, the losses of both a type A object and an antenna should be used. In (32), the integral can be calculated using (25).

For many antennas that are much smaller than the wavelength, the phases of the currents at all points of an antenna and at a port are almost in phase or 180° out of phase. Furthermore, in many cases of small antennas, the phases of the antenna currents and its equivalent currents are almost in phase or are 180° out of phase. Most antennas used in near-field wireless power transfer are very small compared to the wavelength. Therefore, I_1^{at} and I_2^{at} are in phase or 180° out of phase with $\mathbf{J}_1^{\mathrm{eq}}$ and $\mathbf{J}_2^{\mathrm{eq}}$, respectively, in many near-field wireless power transfer systems.

From the gradient of (29), with respect to $\mathrm{Re}(X)$ and $\mathrm{Im}(X)$, we can identify that the maximum power transfer efficiency

$$X = \frac{-\frac{1}{I_1 I_2^{\mathrm{at}}}\int_{R_2^{\mathrm{eq}}} \mathbf{E}_{21}^{\mathrm{eq}} \cdot \mathbf{J}_2^{\mathrm{eq}} dv}{\sqrt{\mathrm{Re}\left(Z_1^{\mathrm{a}} - \frac{1}{I_1 I_1^{\mathrm{at}}}\int_{R_1^{\mathrm{eq}}} \mathbf{E}_{11}^{\mathrm{eq}} \cdot \mathbf{J}_1^{\mathrm{eq}} dv\right) \mathrm{Re}\left(Z_2^{\mathrm{a}} - \frac{1}{I_2 I_2^{\mathrm{at}}}\int_{R_2^{\mathrm{eq}}} \mathbf{E}_{22}^{\mathrm{eq}} \cdot \mathbf{J}_2^{\mathrm{eq}} dv\right)}} \tag{31}$$

$$X = \frac{-e^{-j\left(\arg\left(I_1^{\mathrm{at}}\right)+\arg\left(I_2^{\mathrm{at}}\right)\right)}\int_{R_2^{\mathrm{eq}}} \mathbf{E}_{\mathrm{eob}}^{\mathrm{eq},1} \cdot \mathbf{J}_2^{\mathrm{eq}} dv}{\sqrt{2\frac{P_1^{\mathrm{rad}}}{\eta_1^{\mathrm{rad}}} - \mathrm{Re}\left(e^{-j2\arg\left(I_1^{\mathrm{at}}\right)}\int_{R_1^{\mathrm{eq}}} \mathbf{E}_{\mathrm{ob}}^{\mathrm{eq},1} \cdot \mathbf{J}_1^{\mathrm{eq}} dv\right)}\sqrt{2\frac{P_2^{\mathrm{rad}}}{\eta_2^{\mathrm{rad}}} - \mathrm{Re}\left(e^{-j2\arg\left(I_2^{\mathrm{at}}\right)}\int_{R_2^{\mathrm{eq}}} \mathbf{E}_{\mathrm{ob}}^{\mathrm{eq},2} \cdot \mathbf{J}_2^{\mathrm{eq}} dv\right)}} \tag{32}$$

increases as the real or imaginary part of X increases. Therefore, if the field patterns of the antennas are the same, the maximum power transfer efficiency becomes larger as the radiation efficiencies of the antennas become larger. Thus, making an antenna with a high radiation efficiency is very important for efficient wireless power transfer.

From (29) and (32), we can identify that to calculate the maximum power transfer efficiency, the near-field patterns, equivalent currents, radiated power, and radiation efficiencies of the antennas in transmitting mode are required. The equivalent current and radiated power can be found from the near-field pattern generated when an antenna with a type A object operates in transmitting mode. Therefore, in the case where the antennas are electrically small and antennas and type B objects are electrically not very close, the maximum power transfer efficiency of wireless power transfer can be calculated using only the near-field patterns and the radiation efficiencies of the antennas in transmitting mode which are alone with a type A object. Even when the detailed geometry of the antennas is not known, the maximum power transfer efficiency can be calculated if the near-field patterns and radiation efficiencies of the antennas are given.

If the equivalent currents are simple, i.e., composed of a few point sources, the maximum power transfer efficiency can be calculated relatively rapidly using equivalent currents. We can simply estimate the efficiency of the wireless power transfer if we can guess the simple equivalent currents of the antennas.

B. Optimum Load Impedance

The power transfer efficiency depends on the load impedance. The load impedance that maximizes the power transfer efficiency is called the optimum load impedance. In this section, a method for calculating the optimum load impedance ($Z_{\mathrm{L}}^{\mathrm{opt}}$) is presented. In the case where a source is connected to the port of antenna 1 and a load is connected to the port of antenna 2, the optimum load impedance is as follows [5]:

$$\mathrm{Re}\left(Z_{\mathrm{L}}^{\mathrm{opt}}\right) = \sqrt{\mathrm{Re}(Z_{22})^2 - \frac{\mathrm{Re}(Z_{22})}{\mathrm{Re}(Z_{11})}\mathrm{Re}(Z_{12}Z_{21}) - \frac{\mathrm{Im}(Z_{12}Z_{21})^2}{4\mathrm{Re}(Z_{11})^2}} \tag{33}$$

$$\mathrm{Im}\left(Z_{\mathrm{L}}^{\mathrm{opt}}\right) = \frac{\mathrm{Im}(Z_{12}Z_{21})}{2\mathrm{Re}(Z_{11})} - \mathrm{Im}(Z_{22}). \tag{34}$$

From (28)

$$Z_{12}Z_{21} = \mathrm{Re}(Z_{11})\mathrm{Re}(Z_{22})X_1X_2. \tag{35}$$

Substituting (35) into (33) and (34), $\mathrm{Re}(Z_{\mathrm{L}}^{\mathrm{opt}})$ and $\mathrm{Im}(Z_{\mathrm{L}}^{\mathrm{opt}})$ become

$$\mathrm{Re}\left(Z_{\mathrm{L}}^{\mathrm{opt}}\right) = \mathrm{Re}(Z_{22})\sqrt{1 - \mathrm{Re}(X_1X_2) - \frac{1}{4}\mathrm{Im}(X_1X_2)^2} \tag{36}$$

$$\mathrm{Im}\left(Z_{\mathrm{L}}^{\mathrm{opt}}\right) = \frac{1}{2}\mathrm{Re}(Z_{22})\mathrm{Im}(X_1X_2) - \mathrm{Im}(Z_{22}). \tag{37}$$

If antennas and objects are reciprocal, then $\mathrm{Re}(Z_{\mathrm{L}}^{\mathrm{opt}})$ and $\mathrm{Im}(Z_{\mathrm{L}}^{\mathrm{opt}})$ are

$$\mathrm{Re}\left(Z_{\mathrm{L}}^{\mathrm{opt}}\right) = \mathrm{Re}(Z_{22})\sqrt{1 - \mathrm{Re}(X^2) - \frac{1}{4}\mathrm{Im}(X^2)^2} \tag{38}$$

$$\mathrm{Im}\left(Z_{\mathrm{L}}^{\mathrm{opt}}\right) = \frac{1}{2}\mathrm{Re}(Z_{22})\mathrm{Im}(X^2) - \mathrm{Im}(Z_{22}). \tag{39}$$

To calculate the optimum load impedance, X and Z_{22} are required. X can be calculated from the near-field patterns and radiation efficiencies of antennas 1 and 2 in transmitting mode, and Z_{22} can be calculated from the near-field pattern and the input impedance of antenna 2 in transmitting mode.

VII. Validation and Example

A. Three Dipole Antennas on an Infinite Dielectric

To validate the formulas in Section IV, we calculated the impedance parameters of multiple antennas using the theory presented in Section IV and using the electromagnetic simulator FEKO. We then compared the results obtained with the two methods. In the simulation, three dipole antennas and one linear wire are on the surface of an infinite dielectric. Half of the space is free space and the rest is dielectric. The dielectric constant and the loss tangent of the dielectric are 10 and 0.1, respectively. The lengths of the dipole antennas are 15, 20, and 25 cm, and the length of the linear wire is 30 cm. The diameters of all dipole antennas and the linear wire are 0.1 mm. The conductivity of the material constituting antennas and wire is 2×10^5 S/m. We set the diameters of the antennas and wire and the conductivity of the material to be very small to make the loss large. Three dipole antennas are fed at their centers. The antennas and wire are arranged in the order of the 25 cm dipole antenna, 30 cm linear wire, 15 cm dipole antenna, and 20 cm dipole antenna. The first antenna is the 25 cm dipole antenna (antenna 1), the second antenna is the 15 cm dipole antenna (antenna 2), and the third antenna is the 20 cm dipole antenna (antenna 3). The three dipole antennas

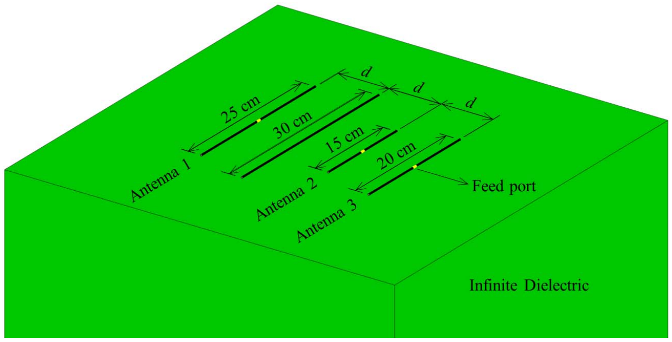


Fig. 8. Simulation configuration for Section VII-A.

and the linear wire are parallel, and the line connecting the centers of the three dipole antennas and the linear wire is perpendicular to the three dipole antennas and one linear wire (Fig. 8). In this section, we set an infinite dielectric to be a type A object and the 30 cm linear wire to be a type B object.

Z_{31} and Z_{22} at frequencies ranging from 100 MHz to 1 GHz were calculated in two cases. In the first case, the distance between the centers of the adjacent two wires (d in Fig. 8) is 1 cm. In the second case, the distance between the centers of the adjacent two wires (d in Fig. 8) is 8 cm. When d is 1 cm, the self-impedance Z_{22} and mutual impedance Z_{31} were calculated using (17) and (12), respectively. The electric field $\mathbf{E}_{22}$ in (17) was calculated for the situation in which the 25 cm dipole antenna and the 20 cm dipole antenna were open-circuited and the 15 cm dipole antenna was excited at its feed port. The electric field $\mathbf{E}_{31}$ in (12) was calculated for the situation in which the 15 cm dipole antenna and the 20 cm dipole antenna were open-circuited and the 25 cm dipole antenna was excited at its feed port. When d is 8 cm, the mutual impedance Z_{31} was calculated using (12) and (15). The electric field $\mathbf{E}_{31}$ was calculated for the situation in which $\mathbf{J}_1^{\text{at}}$ and the 30 cm wire were placed on the surface of an infinite dielectric such that the distance between $\mathbf{J}_1^{\text{at}}$ and the 30 cm wire is 8 cm and the antennas were not present. In the calculation of the mutual impedance for d of 8 cm, the 15 cm dipole antenna was neglected because open-circuited 15 cm dipole antennas rarely scatter. For both cases ($d = 1$, $d = 8$), the current densities $\mathbf{J}_1^{\text{at}}$, $\mathbf{J}_2^{\text{at}}$, and $\mathbf{J}_3^{\text{at}}$ were calculated when the 25 cm dipole antenna, 15 cm dipole antenna, and 20 cm dipole antenna were alone in the half-space, respectively.

Fig. 9 shows the Z_{22} and Z_{31} values for a d of 1 cm calculated using the method of moments in FEKO[3] and those calculated using the formulas presented in Section IV. In Fig. 9, the values calculated with FEKO and those calculated with the theory are almost the same. When d is 1 cm, if the mutual impedance is calculated using (12) and (15) (the same method as that used for a d of 8 cm), the errors between the values calculated with FEKO and with the theory are large. Fig. 10(a) shows the Z_{22} values for a d of 8 cm and the input impedance of the 15 cm dipole antenna that is alone in the half-space. Both are calculated using the method of moments in FEKO. Fig. 10(b) shows the Z_{31} values for a

[3]Impedance parameters were calculated from the scattering parameters obtained with FEKO.

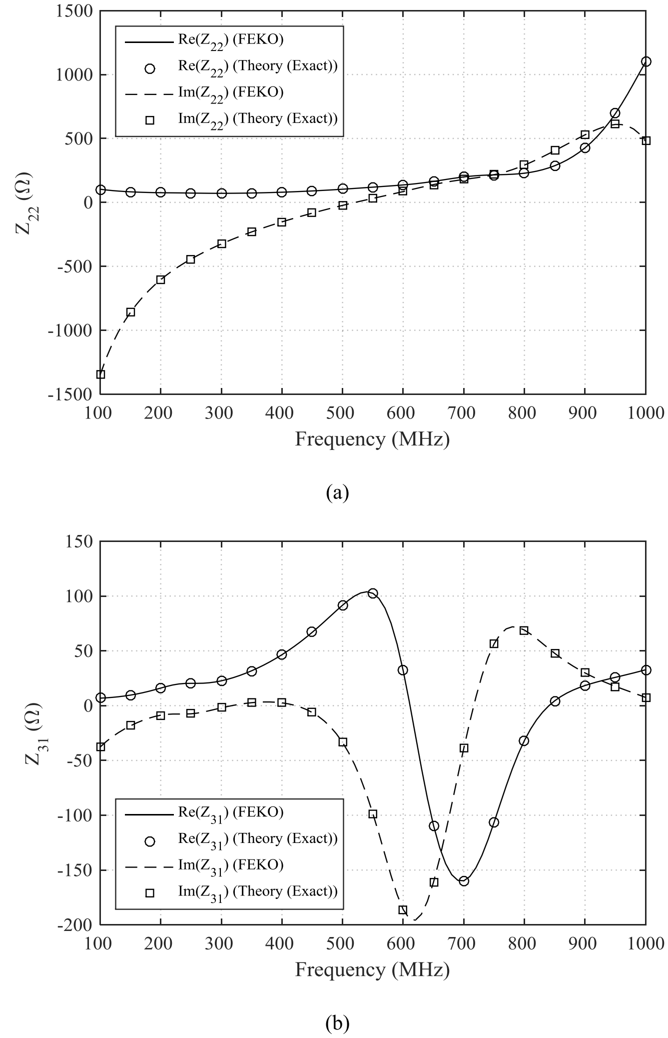
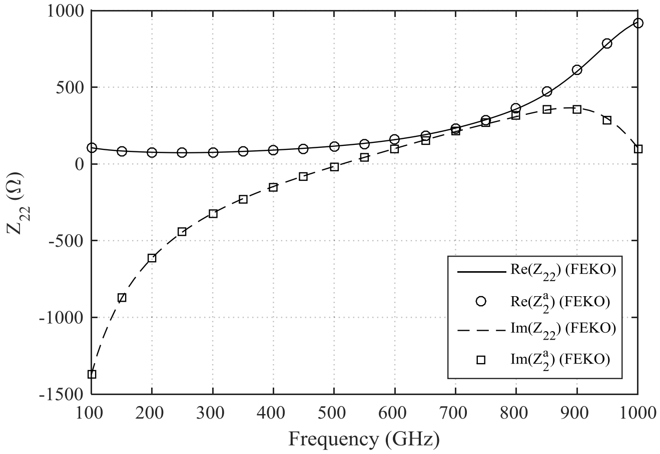


Fig. 9. Z_{22} and Z_{31} when d is 1 cm for the scenario in Fig. 8 calculated with FEKO and calculated exactly with the theory in Section IV. (a) Real and imaginary parts of Z_{22}. (b) Real and imaginary parts of Z_{31}.

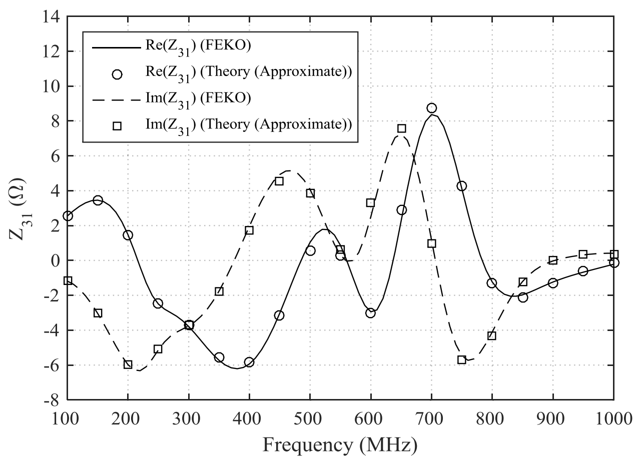
d of 8 cm calculated using FEKO and those approximately calculated using the theory in Section IV. Fig. 10(a) shows that Z_{22} is similar to the input impedance of the 15 cm dipole antenna in the half-space. In Fig. 10(b), we can identify that the mutual impedance can be calculated with a small error using only the current distribution of the transmitting antenna that is alone in the half-space.

B. Two Helical Antennas Above an Infinite Dielectric

We calculated the impedance parameters of two helical antennas over half-space using equivalent currents and the electromagnetic simulator FEKO. Half of the space was free space and rest was a dielectric. The dielectric constant was 10, and the loss tangent of the dielectric was 0.1. The antennas were termed helical antenna 1 and helical antenna 2. For helical antenna 1, the radius was 8 cm, the height was 10 cm, the number of turns was 8, and the diameter of the cross-section of the wire was 1 mm. For helical antenna 2, the radius was 6.8 cm, the height was 12 cm, the number of turns was 10, and the diameter of the cross section of the wire was 1 mm. Both helical antenna 1 and helical antenna 2



(a)



(b)

Fig. 10. Z_{22} and Z_{31} when d is 8 cm for the scenario in Fig. 8 calculated with FEKO and calculated approximately with the theory in Section IV. (a) Real and imaginary parts of Z_{22} and real and imaginary parts of the input impedance of a 15 cm dipole antenna in half-space. (b) Real and imaginary parts of Z_{31}.

were made of copper. The two helical antennas were fed at the middle of the wire, i.e., the lengths of the wires at the two sides of the port were the same. The resonant frequencies of helical antenna 1 and 2 were 29.4 and 30.0 MHz, respectively. The axes of the helical antennas were perpendicular to the dielectric surface. The shortest distance between the center of the helical antenna and the surface of the dielectric was 20 cm for both the helical antennas. The distance between the centers of the two helical antennas was 40 cm (Fig. 11). In this section, we set the infinite dielectric as a type B object, and there is no type A object.[4]

The equivalent current can be found from the coefficients of spherical waves generated by an isolated transmitting antenna in free space. Fig. 12 shows the point source, dipole source, and quadrupole source. One point electric source generates only the TM_{01} mode, and one point magnetic source generates only the TE_{01} mode [8, p. 287] (a point electric

[4]We can also think that a type A object is made of materials that are the same as free space and a type A object is present anywhere outside of the helical antennas and dielectric.

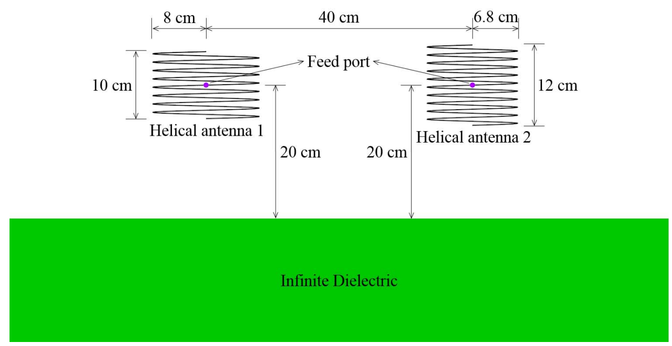


Fig. 11. Simulation configuration for Section VII-B.

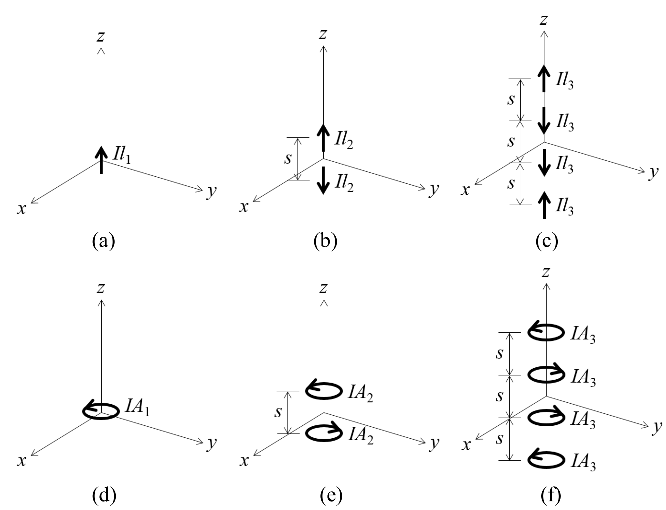


Fig. 12. (a) Point electric source. (b) Dipole electric source. (c) Quadrupole electric source. (d) Point magnetic source. (e) Dipole magnetic source. (f) Quadrupole magnetic source. The centers of the infinitesimal electric dipole and infinitesimal electric loop are on the z-axis. The infinitesimal electric dipoles are perpendicular to the xy plane, and the infinitesimal electric loops are parallel to the xy plane. The center of the source is located at the coordinate origin for all six sources. The distance s between infinitesimal electric dipoles or infinitesimal electric loops in the dipole and quadrupole sources is much smaller than the wavelength. Il_1, Il_2, and Il_3 are the values of the electric moment of an infinitesimal electric dipole in point, dipole, and quadrupole electric sources, respectively. IA_1, IA_2, and IA_3 are the values of the magnetic moment of an infinitesimal electric loop in point, dipole, and quadrupole magnetic sources, respectively.

source is equivalent to an infinitesimal electric dipole, and a point magnetic source is equivalent to an infinitesimal electric loop). A dipole electric source generates only the TM_{02} mode, and a dipole magnetic source generates only the TE_{02} mode [8, p. 288]. A quadrupole electric source generates TM_{01} and TM_{03} modes, and a quadrupole magnetic source generates TE_{01} and TE_{03} modes [8, p. 314]. Because helical antennas 1 and 2 mainly generate the TM_{01}, TM_{03}, TE_{01}, and TE_{03} modes, the point electric source, quadrupole electric source, point magnetic source, and quadrupole magnetic source can be the equivalent currents of helical antennas 1 and 2. We determined the electric moment of an infinitesimal electric dipole and the magnetic moment of an infinitesimal electric loop in equivalent current such that the spherical wave coefficients generated by the equivalent current

TABLE I

COEFFICIENTS OF DOMINANT SPHERICAL WAVES FOR HELICAL ANTENNA 1 EXCITED WITH 1 V

Mode \ Frequency	10 MHz	20 MHz	30 MHz
TM_{01}	$-2.299\times10^{-5}j$	$-1.531\times10^{-4}j$	$-1.286\times10^{-4}+4.751\times10^{-3}j$
TE_{01}	$-7.683\times10^{-6}j$	$-1.023\times10^{-4}j$	$-1.290\times10^{-4}+4.765\times10^{-3}j$
TM_{03}	$2.991\times10^{-10}j$	$7.841\times10^{-9}j$	$1.444\times10^{-8}-5.322\times10^{-7}j$
TE_{03}	$4.273\times10^{-11}j$	$2.193\times10^{-9}j$	$5.828\times10^{-9}-2.142\times10^{-7}j$

TABLE II

COEFFICIENTS OF DOMINANT SPHERICAL WAVES FOR HELICAL ANTENNA 2 EXCITED WITH 1 V

Mode \ Frequency	10 MHz	20 MHz	30 MHz
TM_{01}	$-2.422\times10^{-5}j$	$-1.570\times10^{-4}j$	$-1.182\times10^{-1}-9.625\times10^{-2}j$
TE_{01}	$-6.066\times10^{-6}j$	$-7.865\times10^{-5}j$	$-8.885\times10^{-2}-7.232\times10^{-2}j$
TM_{03}	$1.934\times10^{-10}j$	$4.832\times10^{-9}j$	$7.634\times10^{-6}+6.216\times10^{-6}j$
TE_{03}	$1.594\times10^{-11}j$	$7.346\times10^{-10}j$	$1.451\times10^{-6}+1.182\times10^{-6}j$

and helical antenna were the same. In addition to this method using spherical waves, equivalent current can also be found by another method, such as the method described in [9].

We applied 1 V to isolated helical antenna 1 in free space and calculated the coefficients of the spherical waves generated by helical antenna 1 when the origin of the coordinate system was located at the center of the helix, the z-axis coincided with the axis of the helix, and the feed port was on the x-axis. In addition, we applied 1 V to isolated helical antenna 2 in free space and calculated the coefficients of the spherical waves generated by helical antenna 2 when the origin of the coordinate system was located at the center of the helix, the z-axis coincided with the axis of the helix, and the feed port was on the x-axis. The coefficients of the spherical waves were calculated using the spherical wave function in [5]. The coefficients of the dominant spherical waves generated by helical antennas 1 and 2 are presented in Tables I and II, respectively. The equivalent current for the helical antenna was composed of one point electric source, one point magnetic source, one quadrupole electric source, and one quadrupole magnetic source. Because the TM_{02} and TE_{02} modes are not produced by helical antennas 1 and 2, no dipole sources are present in the equivalent current. We set the distance between the two infinitesimal electric dipoles in the quadrupole electric sources and the distance between the two infinitesimal electric loops in the quadrupole magnetic sources to be 1 cm ($s = 1$ cm in Fig. 12). We found the electric moments of the infinitesimal electric dipoles and the magnetic moments of the infinitesimal electric loops from the spherical wave coefficients of the helical antennas. The TM_{01} and TE_{01} modes generated by the quadrupole electric and magnetic sources are much

TABLE III

ELECTRIC AND MAGNETIC MOMENTS OF EQUIVALENT CURRENT FOR HELICAL ANTENNA 1

Parameter \ Frequency	10 MHz	20 MHz	30 MHz
Il_1	$2.453\times10^{-5}j$	$8.169\times10^{-5}j$	$4.576\times10^{-5}-1.690\times10^{-3}j$
IA_1	$3.912\times10^{-5}j$	$1.303\times10^{-4}j$	$7.299\times10^{-5}-2.696\times10^{-3}j$
Il_3	$-1.699\times10^{-4}j$	$-5.569\times10^{-4}j$	$-3.038\times10^{-4}+1.120\times10^{-2}j$
IA_3	$-1.158\times10^{-4}j$	$-3.715\times10^{-4}j$	$-1.951\times10^{-4}+7.171\times10^{-3}j$
s	1 cm	1 cm	1 cm

TABLE IV

ELECTRIC AND MAGNETIC MOMENTS OF EQUIVALENT CURRENT FOR HELICAL ANTENNA 2

Parameter \ Frequency	10 MHz	20 MHz	30 MHz
Il_1	$2.585\times10^{-5}j$	$8.379\times10^{-5}j$	$4.207\times10^{-2}+3.424\times10^{-2}j$
IA_1	$3.089\times10^{-5}j$	$1.001\times10^{-4}j$	$5.027\times10^{-2}+4.092\times10^{-2}j$
Il_3	$-1.099\times10^{-4}j$	$-3.432\times10^{-4}j$	$-1.606\times10^{-1}-1.308\times10^{-1}j$
IA_3	$-4.322\times10^{-5}j$	$-1.245\times10^{-4}j$	$-4.857\times10^{-2}-3.958\times10^{-2}j$
s	1 cm	1 cm	1 cm

TABLE V

RADIATION EFFICIENCY OF HELICAL ANTENNA 1 AND HELICAL ANTENNA 2

Antenna \ Frequency	10 MHz	20 MHz	30 MHz
Helical antenna 1	0.00647	0.0254	0.0682
Helical antenna 2	0.00835	0.0296	0.0719

smaller than those generated by the point electric and magnetic sources. Therefore, the TM_{01} and TE_{01} modes produced by the quadrupole electric and magnetic sources were ignored in the calculation of the electric and magnetic moments. The electric moment of the infinitesimal electric dipole and the magnetic moment of the infinitesimal electric loop in the equivalent currents for helical antennas 1 and 2 are presented in Tables III and IV, respectively.

We calculated the impedance parameters of two helical antennas over half-space at frequencies from 10 to 30 MHz using (21), (22), (25), and equivalent currents (point sources and quadrupole sources). The current distribution of helical antenna 1 (or 2) over half-space that is excited at its feed port when helical antenna 2 (or 1) is open-circuited is assumed to be similar to the current distribution of helical antenna 1 (or 2) that is alone in free space and excited at its feed port because the antennas and the dielectric are not very close. Therefore, we used $\mathbf{E}_{ob}^{eq,1}$, $\mathbf{E}_{ob}^{eq,2}$, and $\mathbf{E}_{eob}^{eq,1}$ instead of $\mathbf{E}_{11}^{eq}$, $\mathbf{E}_{22}^{eq}$, and $\mathbf{E}_{21}^{eq}$, respectively, in (21), (22), and (25) ($I_1 = I_1^{at}$,

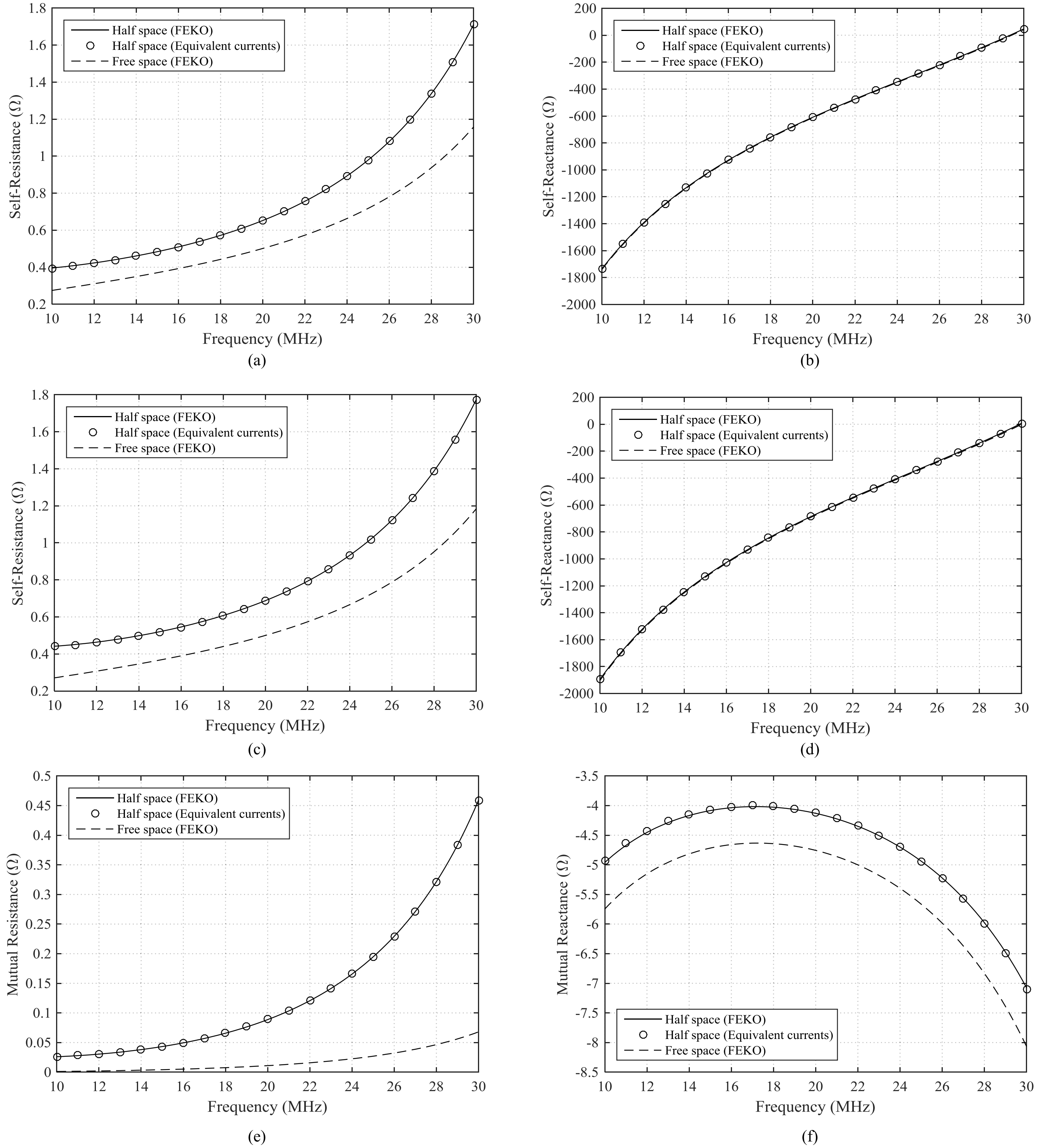


Fig. 13. Impedance parameters of helical antennas in half-space and free space. (a) Real part of Z_{11}. (b) Imaginary part of Z_{11}. (c) Real part of Z_{22}. (d) Imaginary part of Z_{22}. (e) Real parts of Z_{21} and Z_{12}. (f) Imaginary parts of Z_{21} and Z_{12}.

$I_2 = I_2^{\text{at}}$ was set). The electromagnetic fields $\mathbf{E}_{\text{ob}}^{\text{eq},1}$ and $\mathbf{E}_{\text{eob}}^{\text{eq},1}$ were calculated using the Sommerfeld integral [10], [11] in a situation where only $\mathbf{J}_1^{\text{eq}}$ and the infinite dielectric exist. The electromagnetic field $\mathbf{E}_{\text{ob}}^{\text{eq},2}$ was calculated using the Sommerfeld integral in a situation where only $\mathbf{J}_2^{\text{eq}}$ and the infinite dielectric exist. Fig. 13 shows the impedance parameters of helical antennas over half-space calculated using the method of moments in FEKO and equivalent currents. The two results are in good agreement. In Fig. 13, the impedance parameters of the helical antennas in free space calculated using FEKO are also presented. The configuration of the helical antennas in free space is the same as that in half-space, except that the infinite dielectric does not exist.

We also calculated the maximum power transfer efficiency using (29). X in (29) was calculated using two methods. In the first method, X was calculated from (30) using the

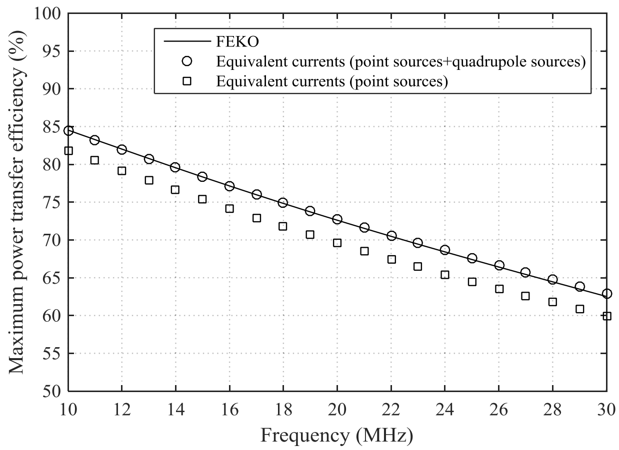


Fig. 14. Maximum power transfer efficiency between helical antennas in half-space.

impedance parameters obtained with FEKO. In the second method, X was calculated using the equivalent currents, radiation efficiencies (radiation efficiencies at the part of the frequencies are presented in TABLE V) and (32). In the second method, we calculated X in two cases. In the first case, X was calculated using the point electric source, point magnetic source, quadrupole electric source, and quadrupole magnetic source. In the second case, X was calculated using the point electric source and point magnetic source. When we calculated X using (32), we used the phase of the current of the point electric source in equivalent current rather than the phase of the port current. That is, we used $\arg(Il_1)$ instead of $\arg(I_1^{\mathrm{at}})$ and $\arg(I_2^{\mathrm{at}})$. Fig. 14 shows the maximum power transfer efficiency between the two helical antennas calculated using FEKO and equivalent currents. Good agreement was found between the results obtained with FEKO and those obtained with the point and quadrupole sources.

VIII. Conclusion

In this paper, we derived formulas that calculate the impedance parameters (Z-parameters) of multiple antennas near objects to analyze wireless power transfer. Furthermore, we proposed a method to calculate the impedance parameters of antennas, the maximum power transfer efficiency, and the optimum load impedance of the wireless power transfer system using equivalent currents that generate the same electromagnetic field as that generated by an antenna in transmitting mode.

The theory developed in this paper facilitates understanding the principles of wireless power transfer. Once the near-field patterns and radiation efficiencies of antennas in transmitting mode are known, the maximum power transfer efficiency between two antennas can be calculated using equivalent currents if the antennas are electrically small and not very close. If the equivalent currents of the antennas are simple, then the maximum power transfer efficiency of wireless power transfer can be predicted simply without using the complex geometry of the antennas.

Although we derived impedance parameters of antennas to analyze wireless power transfer, the formulas for impedance parameters can also be applied to an array antenna. We derived the impedance parameters under the assumption that antennas are fed at an infinitesimal gap. However, the formulas for the impedance parameters are also valid for antennas fed with a coaxial cable.

References

[1] A. Karalis, J. D. Joannopoulos, and M. Soljačić, "Efficient wireless non-radiative mid-range energy transfer," *Ann. Phys.*, vol. 323, no. 1, pp. 34–48, Jan. 2008.

[2] A. P. Sample, D. T. Meyer, and J. R. Smith, "Analysis, experimental results, and range adaptation of magnetically coupled resonators for wireless power transfer," *IEEE Trans. Ind. Electron.*, vol. 58, no. 2, pp. 544–554, Feb. 2011.

[3] T. Imura and Y. Hori, "Maximizing air gap and efficiency of magnetic resonant coupling for wireless power transfer using equivalent circuit and Neumann formula," *IEEE Trans. Ind. Electron.*, vol. 58, no. 10, pp. 4746–4752, Oct. 2011.

[4] J. Lee and S. Nam, "Fundamental aspects of near-field coupling small antennas for wireless power transfer," *IEEE Trans. Antennas Propag.*, vol. 58, no. 11, pp. 3442–3449, Nov. 2010.

[5] Y. G. Kim and S. Nam, "Spherical mode-based analysis of wireless power transfer between two antennas," *IEEE Trans. Antennas Propag.*, vol. 62, no. 6, pp. 3054–3063, Jun. 2014.

[6] E. C. Jordan and K. G. Balmain, *Electromagnetic Waves and Radiating Systems*, 2nd ed. Englewood Cliffs, NJ, USA: Prentice-Hall, 1968.

[7] R. S. Elliott, *Antenna Theory and Design*. Hoboken, NJ, USA: Wiley, 2003, ch. 7.

[8] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*. New York, NY, USA: McGraw-Hill, 1961.

[9] S. M. Mikki and A. A. Kishk, "Quantum particle swarm optimization for electromagnetics," *IEEE Trans. Antennas Propag.*, vol. 54, no. 10, pp. 2764–2775, Oct. 2006.

[10] A. Ishimaru, *Electromagnetic Wave Propagation, Radiation, and Scattering*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1991, ch. 15.

[11] J. R. Wait, *Electromagnetic Waves in Stratified Media*. New York, NY, USA: IEEE Press, 1996.

Yoon Goo Kim (M'12) was born in Gyeonggi-do, South Korea, in 1984. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from Seoul National University, Seoul, South Korea, in 2009, 2011, and 2015, respectively.

Since 2016, he has been involved in the research and development of a radar for the military at a company. His current research interests include electromagnetic theory, wireless power transfer, and array antennas.

Sangwook Nam (S'87–M'88–SM'11) received the B.S. degree in electrical engineering from Seoul National University, Seoul, South Korea, in 1981, the M.S. degree in electrical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 1983, and the Ph.D. degree in electrical engineering from the University of Texas at Austin, Austin, TX, USA, in 1989.

From 1983 to 1986, he was a Researcher with the Gold Star Central Research Laboratory, Seoul. Since 1990, he has been a Professor with the School of Electrical Engineering and Computer Science, Seoul National University. His current research interests include the analysis/design of electromagnetic structures, antennas, and microwave active/passive circuits.