Some Basic Q & A on Wireless Power Transfer

Jongmin Park, Hyunjin Shim, Yoon Goo Kim, Sangwook Nam
Institute of New Media & Communication
Seoul National University
Seoul, Republic of South Korea
city814@ael.snu.ac.kr, hjshim@ael.snu.ac.kr, scika@ael.snu.ac.kr, snam@snu.ac.kr

Abstract— **This paper presents some questions and answers on the wireless power transfer. Firstly, it investigates the physical limitation of a wireless power transfer using spherical modes. It is found that there is a limitation on the transfer efficiency depending on the distance between the resonators and efficiency of antennas. Secondly, we compare the performance of WPTS with respect to the source type. In addition, a method is suggested for efficient WPT when the distance between antennas is varied. Thirdly, a time division method is suggested for WPTS to charge multiple receivers. Finally, we present an analytic formula which can be used to differentiate Inductive Coupling and Magnetic Resonance Coupling which are ambiguously used in wireless power transfer system.**

Keywords— physical limitation; wireless power transfer; frequency tracking; Inductive Coupling(IC); Magnetic Resonance Coupling(MRC); multiple charging

I. INTRODUCTION

Wireless power transfer is a research area of interest recently. There are several analytical models for the analysis of the wireless power transfer characteristics [1], [2]. One of the models is the method using spherical modes [3]. The analysis method using spherical modes gives a clear view of the characteristics of near field coupling between two antennas. This paper shows clearly the physical limitation of wireless power transfer using the spherical mode.

A wireless power transfer system requires high efficiency power transmissions anywhere in the near-field range. However, there are many unresolved issues that remain in the implementation of highly efficient wireless power transfer systems. First of all, it is known that the optimum source and load impedance vary drastically with the coupling distance and the orientation of the antennas [3]. In this paper, we presents a method that makes efficient power transfer possible without simultaneous conjugate matching in cases of varying distance between two antennas [4],[5]. Also, we propose which source type is more efficient. Moreover, we suggest a multiple charging method.

In conventional wireless power transfer systems, the inductive coupling (IC) have been used with proximity to the transmitting antenna. In the case of the magnetic resonance coupling (MRC), the resonant peak of the input impedance is divided into several peaks due to increased mutual inductance between coupled coils [2]. However, these two terms are used ambiguously. In this paper, we extend prior theoretical analysis of WPTS to define two terms; IC and MRC using coupled- mode theory. The key term, critical coefficient is proposed. We take consideration into two sources of WPTS; power and voltage source.

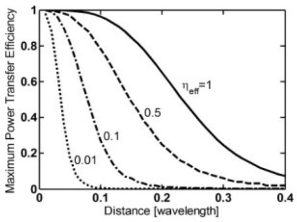

Fig. 1. The maximum power transfer efficiencies of antennas with different radiation efficiencies

II. PHYSICAL LIMITATION OF WIRELESS POWER TRANSFER

We assume that the antennas used in wireless power transfer are the canonical minimum scattering (CMS) antennas and generate only one fundamental spherical mode. A CMS antenna is an antenna that does not scatter electromagnetic fields when its feeding ports are open-circuited. Many antennas that are small relative to wavelength can be modeled as CMS antennas. Because the antennas used for wireless power transfer are very small compared with wavelength, the assumption is reasonable. To derive the Z-parameter between two antennas, we express antennas as the generalized scattering matrix (GSM). The coupling between two antennas can be considered as the cascade of three networks [3]. Two networks are the antennas' GSM and one network represents the space between the two antennas. Z-parameter of two coupled antennas can be obtained by solving the cascaded three networks. The Z-parameter, optimum load impedance and maximum power transfer efficiency when two identical CMS antennas are coupled were given in [3]. When we analyze the maximum power transfer efficiency formula in [3], we can find that the higher the radiation efficiency of an antenna is, the higher the maximum power transfer efficiency is. Fig. 1 shows the maximum power transfer efficiency of a near-field power transfer system comprising two identical CMS antennas for several radiation efficiencies, $_{eff}$. In Fig. 1, the two antennas are at the z-axis of the coordinate system. The maximum power transfer efficiency depends on the radiation efficiency of antennas alone regardless of the specified structure of antennas.

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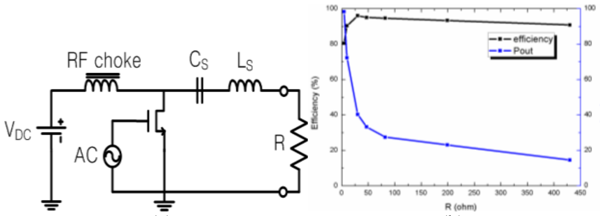


(a) (b)

Fig. 2. The schematic, efficiency and output power of PAs against the load resistance: (a) schematic of Class-D PA (V_{DC}=1V, C_S=58pF, L_S=15uH), (b) efficiency and output power

III. COMPARISON THE SOURCE TYPE

To supply the high power at the source port of WPTS, generally the PA has to be used. To achieve an efficient WPT system despite the variations in impedance, the characteristic of the PA is such that it is insensitive to load variations. Among various PA types, linear PAs, such as Class-A, B, and AB, have limited efficiency [6]. Thus, nonlinear PAs are preferred in this system due to their high efficiency. In a Class-E PA, the theoretical efficiency is determined by the relationship between the optimum shunt capacitance and load impedance. Thus, the impedance variation can cause sharp degradation in the efficiency [7]. On the contrary, the efficiency of the Class-D PA has a characteristic such that it is insensitive to load variations. The overall efficiency can be sustained over large variations in the load. Therefore, the Class-D PA was selected for the validation of the WPTS. Fig. 2 shows the schematic and the efficiency and output power characteristics of the designed class-D amplifier. The input impedance of the measured antenna varied from 4.6 to 400 Ω. It was observed that a high efficiency was maintained over a wide variation in the load resistance.

IV. WPTS OPERATING AT VARYING DISTANCES BETWEEN ANTENNAS

From [1], the optimum load resistance for maximum power transfer efficiency is represented as:

$$R_L^{opt} = R_R \sqrt{1 + \frac{(\omega M)^2}{R_T R_R}} \quad (1)$$

where the subscripts T, R, and L mean the transmitter, the receiver, and the load, respectively. The mutual impedance varies according to the distance between the antennas. Because the optimum load impedance is a function of the mutual impedance, it also varies based on the distance between the antennas. Therefore, it is necessary to simultaneously satisfy the matching condition at both the transmitting and receiving ports in order to achieve maximum power transfer efficiency. However, simultaneous matching is difficult to implement. In this paper, we suggest the methods to solve the matching problem.

A. Frequency Tracking Method

When two resonant antennas are strongly coupled with each other in a near-field region, the resonant frequency is split [8]. The split resonant frequencies are determined by the

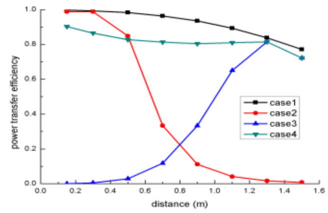


Fig. 3. Comparison of the power transfer efficiency for two coupled small spiral antennas (case 1: fixed frequency with simultaneous matching condition; case 2: frequency tracking with 50 ohm load impedance; case 3: fixed frequency with fixed load impedance (optimum impedance at 1.5 m); case 4: frequency tracking with fixed load impedance (optimum impedance at 1.5 m))

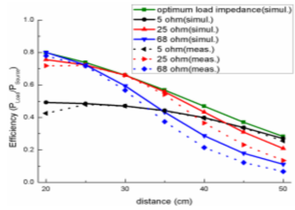


Fig. 4. The total power transfer efficiency for two coupled antennas (line: simulated results including the balun loss and assuming that PA efficiency is 100%, dot: the efficiencies are measured results)

amount of coupling between the antennas. Input impedances at the split resonant frequencies for coupled small antennas have recently been investigated [8]. We noticed that the input impedance at the split resonant frequency is almost equal to the load impedance in the strongly coupled region, provided that the load impedance is much greater than the ohmic resistance and the radiation resistance, as in practical systems. Therefore, it is conceived that input matching and efficient power transfer can be achieved with fixed port impedances by adjusting the frequency of the source to a desired split resonant frequency. Fig. 3 shows the simulation results of the power transfer efficiency several adaptive matching methods. The modified frequency tracking method shows good performance up to the target distance of 1.5 m.

B. Control of Load Resistance

Frequency tracking method is limited when applied to a WPTS. Generally, the relative bandwidth of industrial, scientific, and medical (ISM) bands are less than 1 percent. Therefore, it is easy to violate the frequency regulation. So we investigated a novel method that achieves efficient wireless power transfer when the operating frequency is fixed and the distance is varied. The proposed system only needs to select a proper load resistance when class-D PA is used as a source of the WPTS. The condition of the load resistance and the mutual coupling to maximize the power transfer efficiency is represented as

$$R_R << R_L << \frac{(\omega M)^2}{R_T} \quad (2)$$

The simulated and measured total power transfer efficiency is shown in Fig. 4. The total power transfer efficiency is represented as 5 Ω, 25 Ω, and 68 Ω were the optimum load resistance at 20 cm, 30 cm, and 50 cm, respectively.

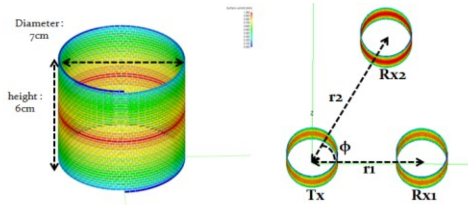


Fig. 5. The maximum power transfer efficiencies of antennas with different radiation efficiencies

$$\eta_{total} = PTE \cdot \eta_{PA} = \frac{P_{Load}}{P_{Source}} \tag{3}$$

Additionally, only the 25 Ω case satisfied the condition of Eq. (2) between 20 and 30 cm. The measured results agreed with the calculated results shown in Fig. 4. When compared with the 5 Ω and 68 Ω cases, the 25 Ω case can efficiently transmit the power in a wide range of distance.

V. WPTS Characteristics for Multiple Receivers by Time Sharing Technique

A. Frequency Characteristics of Multiple Receivers

The method underlying multiple receivers charging remains unclear. Fig. 5 shows the antennas structure and the positions of the multiple receivers. The resonant frequency was 13.56 MHz. The angle between the transmitter and the receivers was set at 60^{o}. The port impedance at the antennas was 50 Ω. Fig. 6 shows the power transfer efficiency (PTE) based on the frequency. The PTE is defined as:

$$\text{PTE}=\frac{\text{P}_L}{\text{P}_{avs}} \tag{4}$$

where P_L is the power delivered to the load, P_{avs} is the power available from the source. The total PTE and the difference between the individual power transfer efficiencies are shown in Fig. 6. Generally, the goal of any power transfer system for multiple receivers is to obtain efficient and equal charging characteristics. At the frequency where the total PTE was efficient, the difference in the PTE of the first and the second receiver was quite high. Therefore, only one of the receivers was well charged, and the others were hardly charged at all. Although the PTE of the first receiver is high when the second receiver is nonexistent, the almost power from the source is transmitted to the second receiver when the coupling between the source and the second receiver is stronger than the coupling between the source and first receiver. The equal charging characteristics can be realized only under the condition that the couplings are same. Such a situation is not advisable.

B. Characteristics of the Time Division WPTS

The analysis at the previous charter highlighted the difficulties surrounding the charging of multiple receivers. To resolve this problem, we propose the time division WPTS. Generally, the WPTS operates at a very low frequency, with the result that the size of the antennas in the system is very small. Therefore, we can assume that an antenna in a WPTS is a quasi-canonical minimum scattering (CMS) antenna.

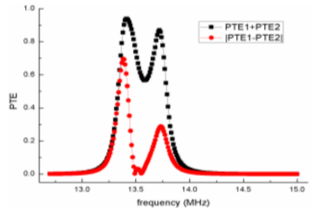
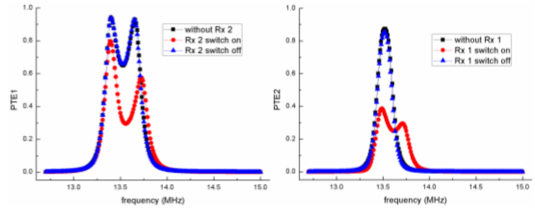


Fig. 6. The maximum power transfer efficiencies of antennas with different radiation efficiencies



(a) (b)

Fig. 7. The maximum power transfer efficiencies of antennas with different radiation efficiencies

A CMS antenna is defined as one that becomes invisible when the antenna port is open-circuited [9]. The switch is added at the port of the receivers. When the switch is off, the load impedance resembles a small capacitor. Therefore, the receiver port is almost open-circuited condition and then the receiver is invisible. As a result, the transmitter can transmit power at the other receiver efficiently. The controller at the receiver controls the switch, which is connected during the allotted time. The PTE of each receiver according to the switch state is shown in Fig. 7. When the switch of the second receiver was off, the PTE was almost same as that of the case of the without the second receiver as shown in Fig. 7(a). Similarly, when the switch of the first receiver was off, the power was efficiently transmitted to the second receiver as shown in Fig. 7(b).

VI. The Coupling of Two Resonator Modes

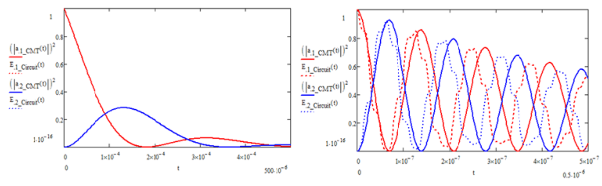
Fig. 8 shows the energy of each resonator as function of time analyzed by Coupled-mode theory and transient circuit theory ($R_{source} = R_{Load} = 0\ \Omega$).
As shown in Fig. 8 (a), in the case of weak-coupling and high Q, the energy flows calculated by each theory match well. However, as in Fig. 8 (b), in the case of strong-coupling and low-Q, the energy flow of transient circuit theory has shorter period than that of Coupled-mode theory. This is because the Q-factor of the equation derived by each theory. The four roots of characteristic equations in S-domain analyzed by transient circuit theory are

$$S_{1,2} = \frac{-\omega_0\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)}{2(1+k_{12})} \pm j\frac{\omega_0\sqrt{4(1+k_{12})-\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)^2}}{2(1+k_{12})},\ S_{3,4} = \frac{-\omega_0\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)}{2(1-k_{12})} \pm j\frac{\omega_0\sqrt{4(1-k_{12})-\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)^2}}{2(1-k_{12})} \tag{5}$$

and those analyzed by Coupled-mode theory are as follows [10]. Parameters of resonators are from [11].

$$S = \frac{-\omega_0\left(\frac{1}{Q}+\frac{1}{Q_{ext}}\right)}{2} \pm j\frac{\omega_0 k_{12}}{2} \tag{6}$$

In Coupled-mode theory, Q-factor of the resonator is fixed at resonant frequency. However, in transient circuit theory



(a) (b)

Fig. 8. Normalized energy flows analyzed by Coupled-mode theory and transient circuit theory in each resonator as function of time: (a) Weak-coupling, High-Q, (b) Strong-coupling, Low-Q

Q-factor is changed over broad frequency band. Even though the energy flows as function of time are more accurate in the case of transient circuit theory analysis, the equations are too complicated and can be applied when two resonators are exactly identical. Whereas the coupled mode theory gives us an approximate solution, it is simple and analytic. So in this paper, Coupled-mode theory analysis is chosen for further investigation.

A. Derivations of Critical coupling coefficient

The term $k_{critical}$ (i.e. critical coupling coefficient) is introduced to discriminate two couplings. In this paper, a criterion is introduced by how much of the energy is received back after the first resonator once transmits energy. If the received energy of the first resonator after one period (1T) is greater than e^{-3} times of the initially transmitted energy, the WPTS system is considered to be using magnetic resonance coupling, and less than 5% of initial value, considered to be under the condition of inductive coupling. The reason of e^{-3} factor is based on the critical coupling between two resonators. The $k_{critical}$ can be obtained when the received energy after one period is as same as e^{-3} times of the initial energy.

$$k_{critical} = \frac{\omega_0\left(\frac{1}{Q_1}+\frac{1}{Q_2}+\frac{1}{Q_{ext_1}}+\frac{1}{Q_{ext_2}}\right)\cdot\pi}{3} \tag{7}$$

B. Derivations of Critical coupling coefficient

To validate the theory, the sizes of resonators are designed as [4]. As in Table I, the results satisfy the condition proposed. In the case of voltage source, $k_{critical}$ using Eq. (7) is 0.068. The simulated critical coupling coefficient is 0.0645. These two $k_{critical}$ have 5.15% error. This is because loaded Q becomes higher as source impedance is ignored.

TABLE I
ENERGY OF FIRST RESONATOR

source	k_{12}	Energy		Type of coupling
		$\|a_1(1T)\|$	$\|a_1(0)\|\cdot e^{-3}$	
Power	0.05	$4.5\cdot10^{-4}$	0.05	IC
	0.128	0.049		Critical
	0.5	0.46		MRC
Voltage	0.05	0.013		IC
	0.068	0.058		Critical
	0.5	0.67		MRC

VII. CONCLUSION

This paper investigates the physical limitation of a wireless power transfer using spherical modes. It shows that the radiation efficiency of an antenna is a single design parameter for wireless power transfer. We do not need to know the specified structure of antennas to analyze the characteristics of wireless power transfer.

We compared several types of PAs as a source of the WPTS. And we proposed a modified frequency tracking method with a complex load matched at the target distance to achieve a stable efficiency beyond the strongly coupled region. Also we proposed WPTS using the class-D PA as a source and using a proper load resistance.

The characteristics of the WPTS to charge the multiple receivers are analyzed. We conclude that in terms of multiple receivers' charging, the proposed time division WPTS can transmit power efficiently and equally.

The Coupled-mode theory and the transient circuit theory are analyzed. Using the Coupled-mode theory, the criterion of inductive coupling and magnetic resonance coupling in WPTS using power and voltage sources is proposed. Using the proposed definition of critical coupling coefficient, these two terms can be clarified analytically.

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