DOA Estimation based on Compressive Sensing using Generalized Scattering Matrix

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***Abstract*—This paper presents a direction of arrival (DOA) estimation method using compressive sensing. The CS model is based on the generalized scattering matrix (GSM), presenting antenna characteristics in a spherical mode analysis including mutual coupling between antennas. As a result, the induced currents on antennas by incident waves can be obtained exactly and an accurate solution can be derived. An example is given with 1×6 uniform linear dipole array antenna. In addition, the output is compared with that of the MUSIC algorithm which is a popular method of DOA estimation.**

I. Introduction

As communication technology has been improved, array antennas have been commonly used for new techniques, such as massive MIMO for high speed and high capacitance [1]. An array antenna which is composed of a number of element antennas, has an inherent problem, namely mutual coupling. [2] It exists when closed spaced antennas are receiving and transmitting electromagnetic waves. Then, element antennas are coupled electromagnetically and they influence each other to change their characteristics, which are an active impedance and active radiation pattern. Therefore, when an array antenna is designed, mutual coupling should be considered to achieve accurate results.

When an array antenna receives propagating waves, currents are induced to each element antenna. The direction of arrival (DOA) estimation is a technique to denote where the waves come from using the induced currents. Because an array antenna is used in a DOA estimation, mutual coupling should be considered as well. Up to now, a number of algorithms have been introduced for DOA estimation, mainly the MUSIC and ESPRIT algorithms [3-6]. As well there are variations to modify mutual coupling [7, 8].

Compressive sensing has also been applied to DOA estimations for spare signals from its introduction [9-11]. There are some techniques for increasing accuracy, however, more effort is needed to consider mutual coupling.

In this paper, a DOA estimation technique based on compressive sensing using the generalized scattering matrix (GSM) is presented. The GSM represents antenna characteristics in a spherical mode. Using the GSM of an element antenna, the overall GSM which shows characteristics of an array antenna in spherical mode could be composed including mutual coupling. By using the overall GSM, the measurement matrix of compressive sensing could be made, as a result, it provides a more accurate DOA estimation result.

In section II, the overview of GSM and the compressive sensing mode using GSM are presented. In section III, simulation results are given using a 1×6 uniform linear dipole array antenna. The result is compared with that of the compressive sensing model without mutual coupling and the MUSIC algorithm which is the most popular DOA algorithm. Finally, section IV, there presents the conclusion.

II. Compressive Sensing Mode Using GSM

A. Overall Generalized Scattering Matrix (GSM)

The GSM represents the characteristics of an antenna in spherical mode [12]. The form is

$$\begin{bmatrix} \rho_i & \mathbf{r}_i \\ \mathbf{t}_i & \mathbf{s}_i - \mathbf{I}_i \end{bmatrix} \begin{bmatrix} v_i \\ \mathbf{a}_i \end{bmatrix} = \begin{bmatrix} w_i \\ \mathbf{b}_i^s \end{bmatrix} \tag{1}$$

And with the geometric information, it is used to compose the overall GSM which expresses characteristics of an array antenna in the form of (2) [13].

$$\begin{bmatrix} \mathbf{\Gamma}_G & \mathbf{R}_G \\ \mathbf{T}_G & (\mathbf{S}_G - \mathbf{I}_G) \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{a}_d \end{bmatrix} = \begin{bmatrix} \mathbf{w} \\ \mathbf{b}^s \end{bmatrix} \tag{2}$$

The parameters are

$$\begin{aligned} \mathbf{\Gamma}_G &= \mathbf{\Gamma} + \mathbf{RG}[\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}]^{-1}\mathbf{T} \\ \mathbf{R}_G &= \mathbf{R} + \mathbf{RG}[\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}]^{-1}(\mathbf{S} - \mathbf{I}) \\ \mathbf{T}_G &= [\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}]^{-1}\mathbf{T} \\ (\mathbf{S}_G - \mathbf{I}_G) &= [\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}]^{-1}(\mathbf{S} - \mathbf{I}) \end{aligned} \tag{3}$$

which are explained more in detail [13].

The advantage of using the GSM is, most importantly, taking account of the mutual coupling effects in an array antenna. Moreover, once the GSM of the antenna element is obtained, it is easy to obtain the overall GSMs of any structures of array antennas composed of antenna elements.

B. Compressive Sensing Model using GSM

The compressive sensing model for DOA estimation is

$$\mathbf{y} = \mathbf{Ax} + \mathbf{e} \tag{4}$$

where $\mathbf{y}$ is the induced currents, $\mathbf{A}$ is the measurement matrix composed of the premeasured induced currents of each angle,

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$\mathbf{x}$ is the sparse signal, and $\mathbf{e}$ is the noise. When the number of element antennas is N_a and the number of angular resolutions for the signal is N_r, the dimension of $\mathbf{y}$ is $(N_a\times 1)$, $\mathbf{A}$ is $(N_s\times N_a)$, and $\mathbf{x}$ and $\mathbf{e}$ are $(Nr\times 1)$.

When $\mathbf{Q}$ is a set of incident signal mode coefficients for an angle, it is described as

$$\mathbf{Q} = \begin{bmatrix} Q_1 & \cdots & Q_j & \cdots & Q_J \end{bmatrix}^T \tag{5}$$

where Q_j is j^{th} mode coefficient. Then, the induced currents for each antenna can be derived using the overall GSM as

$$\mathbf{I}_p = -\frac{\mathbf{R}_G \mathbf{G}_{i,center} \mathbf{Q}}{2\sqrt{\mathrm{Re}(Z_0)}} \tag{6}$$

where $\mathbf{G}_{i,center}$ is the general transition matrix from the center to the i^{th} antenna and Z_0 is the characteristic impedance of an element antenna.

When the reference mode coefficient matrix $\mathbf{Q}_{ref}$ is

$$\mathbf{Q}_{ref} = \begin{bmatrix} \mathbf{Q}_1 & \cdots & \mathbf{Q}_n & \cdots & \mathbf{Q}_{N_r} \end{bmatrix} \tag{7}$$

for each incident angle, the measurement matrix A is

$$\mathbf{A} = -\frac{\mathbf{R}_G \mathbf{G}_{i,center} \mathbf{Q}_{ref}}{2\sqrt{\mathrm{Re}(Z_0)}} \tag{8}$$

Each column of the measurement matrix $\mathbf{A}$ represents the induced currents on antenna elements including mutual coupling.

The measurement matrix $\mathbf{A}$ is composed of imaginary numbers. Therefore, the basis pursuit (BP) which is the basic algorithm of compressive sensing is difficult to be applied. Therefore, to obtain a better result, the basis pursuit denoising (BPDN) in (10) is chosen as the recovery algorithm [14].

$$\text{For some } \lambda\geq 0,\ \min_{\mathbf{x}} \lambda\|\mathbf{x}\|_1 + \|\mathbf{Ax}-\mathbf{y}\|_2^2 \tag{9}$$

III. SIMULATION RESULT

A simulation is performed with a 1×6 uniform linear dipole array example as depicted in Fig. 1. The target frequency is 1.5 GHz. The spacing between antennas d is 0.5 λ (10 cm) and the antenna length l is 0.4 λ (8 cm). The radius is 0.0025 λ (0.05 cm). The antenna material is PEC and the GSM of the antenna element is obtained from the MoM simulation too, FEKO Suite 6.2.

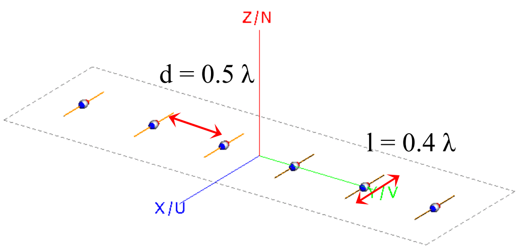

Fig. 1. 1×6 uniform linear dipole array.

An incident wave is assumed a plane wave. When a plane wave is coming from (θ_0, φ_0), its spherical mode expression is given in [12] as

$$\vec{\mathbf{E}}_0 \exp(i\vec{\mathbf{k}}_0\cdot\vec{\mathbf{r}}) = \frac{k}{\sqrt{\eta}}\sum_{j=1}^{J} Q_j \vec{\mathbf{F}}_j^{(1)}(r,\theta,\phi) \tag{10}$$

where Q_j is

$$Q_{smn} = \frac{k}{\sqrt{\eta}}(-1)^m \sqrt{4\pi} i \vec{\mathbf{E}}_0 \cdot \vec{\mathbf{K}}_{s,-m,n}(\theta_0,\phi_0) \tag{11}$$

($\vec{\mathbf{K}}_{s,-m,n}$: far-field wave function, $\vec{\mathbf{k}}_0$: propagation vector, k : wave number, η : intrinsic impedance)

In addition, λ of the BPDN algorithm is chosen as 10^{-7} for the best results.

A. *One incident wave*

When a 10 V/m y-polarized plane wave is coming from (θ_0, φ_0) = (45 °, 270 °) in SNR 20 dB, the estimated result is depicted in Fig. 2 as the first example. The result of the proposed method is compared with those of the MUSIC algorithm with and without mutual coupling. Though the MUSIC algorithm without mutual coupling provides an accurate estimated result, when mutual coupling is considered, the result is worse. However, the proposed DOA estimation method — compressive sensing using the GSM — provides prescise result as that of the MUSIC algorithm without mutual coupling.

B. *Two incident waves of a 40° difference*

The second example is shown in Fig. 3 with two incident waves. There are 10 V/m y-polarized plane waves coming from (θ_0, φ_0) = (25 °, 270 °), (65 °, 270 °) in SNR 20 dB. In this case, compressive sensing using the GSM shows a similar result to that of the MUSIC algorithm without mutual coupling. In addition, it is better than that of the MUSIC algorithm with mutual coupling. Moreover, it estimates the wave from (65 °, 270 °) more precisely than the MUSIC algorithm.

C. *Two incident waves of a 5° difference*

In Fig. 4, the last example is shown with two adjacent incident waves. The incident waves are the same as the second example — 10 V/m y-polarized plane waves — but their incident angles are different: (θ_0, φ_0) = (42 °, 270 °), (47 °, 270 °). The MUSIC algorithm without mutual coupling distinguishes the two adjacent incident waves as though it is somewhat inaccurate. As well, because of the mutual coupling, the MUSIC algorithm with mutual coupling fails to identify the difference between the two incident waves. However, the proposed method — compressive sensing using the GSM — not only distinguishes the two waves, but also provides a precise result.

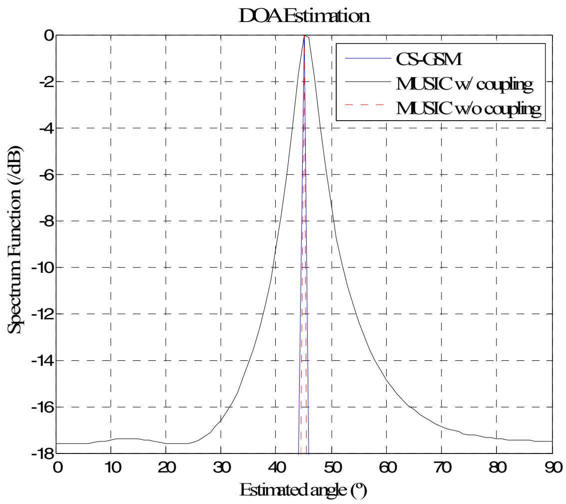

Fig. 2. One incident wave from (θ_0, φ_0) = (45 °, 270 °)

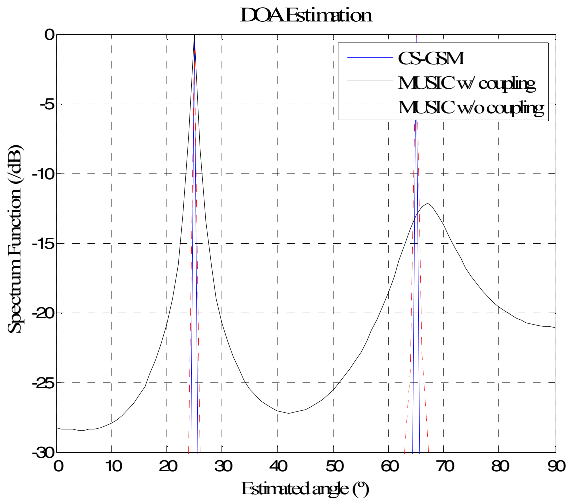

Fig. 3. Two incident waves from (θ_0, φ_0) = (25 °, 270 °), (65 °, 270 °)

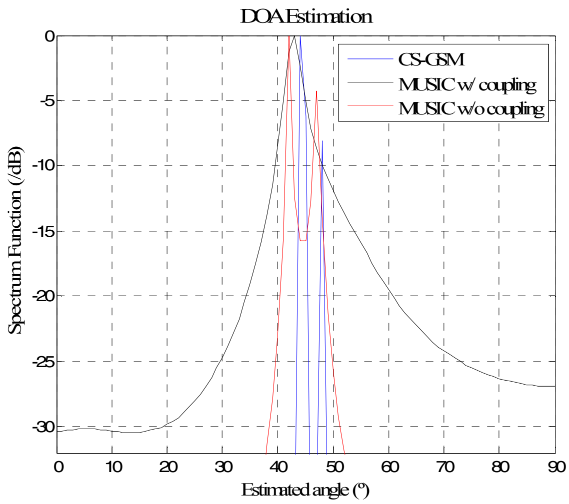

Fig. 4. Two incident waves from (θ_0, φ_0) = (42 °, 270 °), (47 °, 270 °)

IV. CONCLUSION

This paper presents a new method estimating DOA based on compressive sensing. It considers the entire mutual coupling in an array antenna while taking advantage of the GSM. It is examined in three cases. For every case, the results are compared with that of the MUSIC algorithm with and without mutual coupling. Moreover, according to the simulation results, the proposed method provides an outstanding performance. In addition, it is easy to be extended to any other array antenna structure once the GSM of the antenna element is known.

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REFERENCES

[1] Erik G. Larsson, "Massive MIMO for Next Generation Wireless Systems", Communication Magazine, IEEE, vol.52, no.2, pp. 186-195, Feb. 2014

[2] Inder J. Gupta, Aharon A. Ksienski, "Effect of Mutual Coupling on the Performance of Adaptive Arrays", IEEE Trans. Antenna Propag., vol.31, no.5, pp. 785-791, Sep. 1983.

[3] Ralph O. Schmitdt, "Multiple Emitter Location and Signal Parameter Estimation", IEEE Trans. Antenna Propag., vol. AP-34, no.3, Mar. 1986

[4] Fei Wen, Qun Wan, Rong Fan, Hewen Wei, "Improved MUSIC Algorithm for Multiple Noncoherent Subarrays", IEEE Letter. Signal Processing, vol.21, no.5, May 2014.

[5] R. Roy and T. Kailath, "ESPRIT – Estimation of signal parameters via rotational invariance techniques," IEEE Trans. Acoust. Speech, Signal

[6] Yaunming Guo, Wei Li, yanyan Zuo, Junyuan Shen, "A Nyström-Based Esprit Algorithm for DOA Estimation of Coherent Signals", Antennas and Propagation (ISAP), 2013 Proceedings of the International Symposium on, vol.1, Oct. 2013

[7] Lui, H. S. and H. T. Hui, "Effective mutual coupling compensation for direction-of-arrival estimations using a new, accurate determination method for the receiving mutual impedance," Journal of Electromagnetic Waves and Applications, vol. 24, no. 2-3, 271-281, 2010.

[8] H. Rogier and E. Bonek, "Analytical spherical-mode based compensation of mutual coupling in uniform circular arrays for direction-of-arrival estimation," Int. J. Electr. Comm., vol. 60, no. 3, pp. 179–189, Mar. 2006

[9] I. Bilik, "Spatial compressive sensing for direction-of-arrival estimation of multiple sources using dynamic sensor arrays," Trans. Aerosp. Electron. Syst., vol.47, no.3, pp. 1754–1769, 2011.

[10] M. Carlin, P. Rocca, G. Oliveri, F. Viani, and A. Massa, "Directions-of arrival estimation through Bayesian compressive sensing strategies," IEEE Trans. Antennas Propag., vol. 61, no. 7, pp. 3828–3838, 2013.

[11] Fortunati, S., Grasso, R., Gini, F., Gerco, M. S., "Single snapshot DOA estimation using compressed sensing", Acoustics, Speech and Signal Processing (SCASSP), 2014 IEEE International Conference on, pp.2297-2301, May, 2014

[12] J. E. Hansen, *Spherical Near-field Antenna Measurements*, Peter Peregrinus Ltd., 1988.

[13] Jesús Rubio, Miguel A. González, and Juan Zapata, "Generalized-scattering-matrix analysis of a class of finite arrays of coupled antennas by 3-D FEM and spherical mode expansion", IEEE Trans. Antennas Propag., vol.53, no.3, pp.1133-1144, Mar. 2005

[14] Simon Foucart, Holger Rauhut, A Mathematical Introduction to Compressive Sensing, Birkhauser, 2010.