Mode-Based Computation Method of Channel Characteristics for a Near-Field MIMO

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***Abstract*—This letter proposes a new mode-based method for estimating the channel characteristics of a near-field multiple-input–multiple-output (MIMO). When an antenna array composed of a short electric dipole and a small solenoidal loop is used, a 2×2 near-field MIMO can be effectively constructed according to the orthogonality of the TM_{10} and TE_{10} modes. By using the proposed method, the transmission characteristics of the MIMO and the mutual coupling effect at the antenna array can be easily calculated. In addition, the capacity of the MIMO can also be calculated by using the analysis results.**

***Index Terms*—Addition theorem, antenna mutual coupling, multiple-input–multiple-output (MIMO) channel, near-field MIMO, small antennas.**

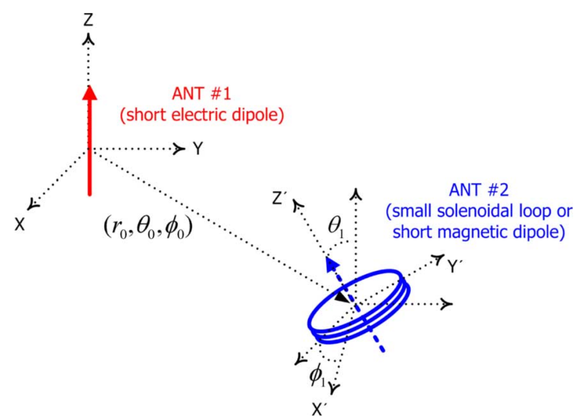

Fig. 1. Antenna array consisting of a short electric dipole and a small loop.

I. Introduction

VARIOUS communication technologies have been proposed to support the explosive growth of information transfer. In particular, multiple-input–multiple-output (MIMO) technology, which uses multiple antennas for transmitting and receiving signals, has been developed to increase the maximum available data rate for limited wireless resources. According to MIMO theory, a wireless channel can be spatially parallelized by using transmitting and receiving arrays, and thus the capacity of the channel can be increased by using space–time multiplexing [1].

In a conventional MIMO, where transmitting and receiving arrays are placed in the far-field region of the antenna elements, a channel can be multiplexed by the rich multipath environment. The channel characteristics are therefore determined by the scattering properties of the channel. In a near-field MIMO, the transmitting and receiving arrays are placed in the near-field region, where the reactive coupling dominates when compared to far-field multipath scattering of classical MIMO [2]. In a previous study, the channel characteristics of near-field MIMO were analyzed by a full-wave simulation, and it was shown that each eigenmode of the antenna array could be considered as an individual independent channel [3].

If an electric short dipole and a small solenoidal loop (or magnetic short dipole) are used for transmitting and receiving arrays, the near-field 2×2 MIMO can be easily constructed because the spherical modes generated by the antenna can be considered as independent multiplexed channels. The analysis for the near-field 2×2 MIMO, which comprises an electric dipole and a $\pi/2$-tilted magnetic short dipole, has recently been presented [4]. However, this method is difficult to apply in a general case where each element of the transmitting and receiving arrays is arbitrarily placed with a different orientation and distance.

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II. Analysis of Transmission Characteristics of Near-Field MIMO Comprising Small Antennas

The electromagnetic field in the space around an antenna can be described as a combination of orthogonal spherical waves. According to the addition theorem, the spherical waves that spread out from the transmitting antennas can be transformed into another combination of incoming spherical waves at the receiving antenna. Hence, the interaction between the transmitting and receiving antennas can be estimated by using the addition theorem [5]–[7]. If the antennas are electrically small, the interaction between the transmitting and receiving antennas can be simplified because electrically small antennas predominantly generate a TM_{10} or TE_{10} mode. A number of studies based on the addition theorem has recently been conducted regarding the mutual impedance analysis between small antennas [5]–[7].

If transmitting and receiving antenna arrays are composed of a short electric dipole and a small solenoidal loop as shown in Fig. 1 and the relative positions of antennas are given as shown in Fig. 2, the equivalent circuit for describing transmissions between the transmitting and receiving arrays can be given as in Fig. 3, based on the equivalent circuit description as shown in [6] and [7]. Because a short electric dipole generates only a TM_{10} mode and a small solenoidal loop (or a short magnetic dipole) dominantly generates only a TE_{10} mode, the transmitted

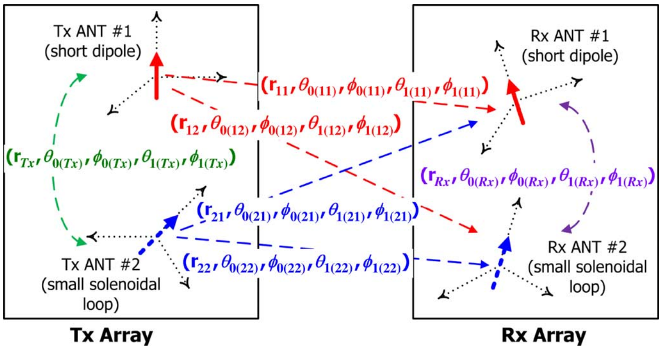

Fig. 2. Near-field 2×2 MIMO using an antenna array comprising a short electric dipole and a small solenoidal loop (or a short magnetic dipole).

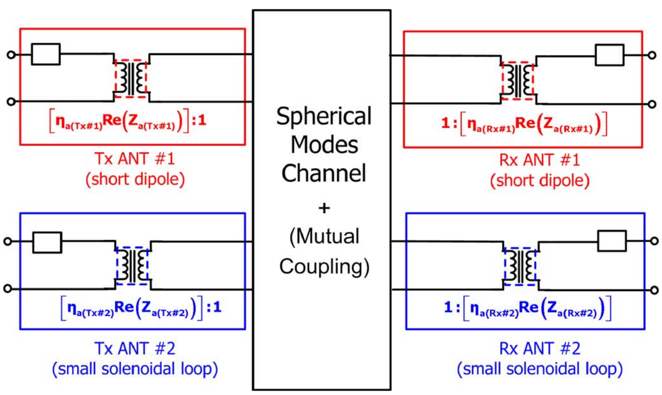

Fig. 3. Equivalent circuit description of near-field 2×2 MIMO.

signals from each transmitting antenna can be simply described by a single outgoing spherical wave, defined as

$$a_{\mathrm{TM}_{10}(\mathrm{Tx}\#1)} = \frac{V_{\mathrm{a(Tx\#1)}}}{\sqrt{\mathrm{Re}\left(Z_{\mathrm{a(Tx\#1)}}\right)}}$$
$$a_{\mathrm{TE}_{10}(\mathrm{Tx}\#2)} = \frac{V_{\mathrm{a(Tx\#2)}}}{\sqrt{\mathrm{Re}\left(Z_{\mathrm{a(Tx\#2)}}\right)}} \tag{1}$$

and similarly, the received signals at each receiving antenna can be described simply by a single incoming spherical wave, defined as

$$b_{\mathrm{TM}_{10}(\mathrm{Rx}\#1)} = \frac{V_{\mathrm{a(Rx\#1)}}}{\sqrt{\mathrm{Re}\left(Z_{\mathrm{a(Rx\#1)}}\right)}}$$
$$b_{\mathrm{TE}_{10}(\mathrm{Rx}\#2)} = \frac{V_{\mathrm{a(Rx\#2)}}}{\sqrt{\mathrm{Re}\left(Z_{\mathrm{a(Rx\#2)}}\right)}} \tag{2}$$

where the terminal voltage and impedance of each antenna are given as V_{a} and Z_{a}, respectively.

Based on the analysis in [6] and [7], which is derived from the addition theorem, the transmission matrix of a near-field 2×2 MIMO comprising short electric and magnetic dipoles can be given as (3), shown at the bottom of the page, with each parameter given as

$$\begin{aligned} &A'_{10,10}(r_0, \theta_0, \phi_0, \theta_1, \phi_1) \\ &\quad = \cos\theta_1 \left[P_0(\cos\theta_0) h_0^{(2)}(kr_0) + P_2(\cos\theta_0) h_2^{(2)}(kr_0)\right] \\ &\quad - \frac{1}{2}\sin\theta_1 P_2^1(\cos\theta_0) h_2^{(2)}(kr_0)(\cos\phi_1\cos\phi_0 + \sin\phi_1\sin\phi_0) \end{aligned} \tag{4}$$

$$\begin{aligned} &B'_{10,10}(r_0, \theta_0, \phi_0, \theta_1, \phi_1) \\ &\quad = \frac{3}{2}\sin\theta_1 P_1^1(\cos\theta_0) h_1^{(2)}(kr_0)(\cos\phi_1\sin\phi_0 - \sin\phi_1\cos\phi_0) \end{aligned} \tag{5}$$

where k is the propagation constant, η_{a} is the efficiency of each antenna, $h_n^{(2)}(x)$ is the spherical Hankel function of the second kind, and $P_n^m(x)$ is the associated Legendre function of the first kind. Accordingly, the channel matrix of the near-field 2×2 MIMO can be given as (6) by using the relations between the spherical wave and the terminal voltage shown in (1) and (2)

$$\mathbf{H}_{\omega} = \begin{bmatrix} \frac{1}{\sqrt{\mathrm{Re}(Z_{\mathrm{a(Rx\#1)}})}} & 0 \\ 0 & \frac{1}{\sqrt{\mathrm{Re}(Z_{\mathrm{a(Rx\#2)}})}} \end{bmatrix}^{-1} \times \mathbf{T} \begin{bmatrix} \frac{1}{\sqrt{\mathrm{Re}(Z_{\mathrm{a(Tx\#1)}})}} & 0 \\ 0 & \frac{1}{\sqrt{\mathrm{Re}(Z_{\mathrm{a(Tx\#2)}})}} \end{bmatrix}. \tag{6}$$

Because each antenna element of the array shown in Fig. 1 generates different spherical modes, the channel can be described as multiplexed based on the orthogonality of the spherical modes. Therefore, each spherical mode can be considered an independent channel. Because the cases that use TE_{10} and TM_{10} modes have orthogonal polarizations with the same radiation pattern, the channel can be said to be multiplexed by the polarization diversity.

III. Mutual Coupling Considerations

If the antenna arrays are implemented in a restricted space, the antenna elements are placed close to each other and then mutually coupled. In a transmitting array, the radiated field generated from a certain antenna can also be received by another antenna element of the array. The actual voltage applied in the antenna, therefore, is altered from the value initially given. If a transmitting array composed of n antennas is given, the voltage

$$\mathbf{T} = \frac{1}{2}\begin{bmatrix} \sqrt{\eta_{\mathrm{a(Tx\#1)}}\eta_{\mathrm{a(Rx\#1)}}} A'_{10,10}\left(r_{0_{(11)}}, \theta_{0_{(11)}}, \phi_{0_{(11)}}, \theta_{1_{(11)}}, \phi_{1_{(11)}}\right) & \sqrt{\eta_{\mathrm{a(Tx\#2)}}\eta_{\mathrm{a(Rx\#1)}}} B'_{10,10}\left(r_{0_{(12)}}, \theta_{0_{(12)}}, \phi_{0_{(12)}}, \theta_{1_{(12)}}, \phi_{1_{(12)}}\right) \\ \sqrt{\eta_{\mathrm{a(Tx\#1)}}\eta_{\mathrm{a(Rx\#2)}}} B'_{10,10}\left(r_{0_{(21)}}, \theta_{0_{(21)}}, \phi_{0_{(21)}}, \theta_{1_{(21)}}, \phi_{1_{(21)}}\right) & \sqrt{\eta_{\mathrm{a(Tx\#2)}}\eta_{\mathrm{a(Rx\#2)}}} A'_{10,10}\left(r_{0_{(22)}}, \theta_{0_{(22)}}, \phi_{0_{(22)}}, \theta_{1_{(22)}}, \phi_{1_{(22)}}\right) \end{bmatrix} \tag{3}$$

vector $\mathbf{v}$, which is actually applied at each antenna element, can be found by

$$\mathbf{v} = \mathbf{C_T}\mathbf{Z^T}(\mathbf{Z^T} + \mathbf{Z_s})^{-1}\mathbf{v_s} = \mathbf{C_{TX}}\mathbf{v_s} \tag{7}$$

where the vector of source voltages is given as $\mathbf{v_s}$, $\mathbf{Z^T}$ is the impedance matrix at the transmitting end, $\mathbf{Z_s}$ is the diagonal matrix composed of the source impedances Z_{sm} of each transmitter, and d $\mathbf{C_T}$ is the diagonal matrix with entries $[C_T]_{nn} = (Z_{nn}^T + Z_{sn})/Z_{nn}^T$ [8].

When electromagnetic fields are received at the receiving antennas, the received fields at each antenna element can be reradiated and received by another receiving antenna of the array. Through this process, the final received voltage at the terminal can be changed. Recently, a new study about the mutual coupling in the receiving mode has shown that the coupling matrix at the receiving array needs to be given differently because the coupling paths in the receiving mode differ from those in the transmitting mode [9]. If a receiving array is composed of n antennas and the terminal voltage vector generated by the initially received fields is given as $\mathbf{x}$, the received terminal voltage vector $\mathbf{v_r}$, including mutual coupling effects, is given as

$$\mathbf{v_r} = \begin{bmatrix} 1 & \frac{-Z_{12}}{Z_{l1}} & \cdots & \frac{-Z_{1n}}{Z_{l1}} \\ \frac{-Z_{21}}{Z_{l2}} & 1 & \cdots & \frac{-Z_{2n}}{Z_{l2}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-Z_{n1}}{Z_{ln}} & \frac{-Z_{n2}}{Z_{ln}} & \cdots & 1 \end{bmatrix} \mathbf{x} = \mathbf{C_{RX}}\mathbf{x} \tag{8}$$

where the load impedances of each receiver are given as Z_{lm} and the mutual impedance between the ith and jth antennas is given as Z_{ij}.

IV. Channel Matrix for 2 × 2 Near-Field MIMO and Capacity Analysis

In [5] and [6], the mutual impedance of two small coupled antennas is shown to be easily estimated by using a mode-based analysis. Hence, if the transmitting and receiving antenna arrays are made up as shown in Fig. 1 and sufficiently separated from each other so that the antenna spheres of the antennas do not overlap, mutual impedance can be given as [7]

$$\begin{aligned} Z_{12(\mathrm{Tx})} &= Z_{21(\mathrm{Tx})} \\ &= \sqrt{\eta_{\mathrm{a(Tx\#1)}}\mathrm{Re}\left(Z_{\mathrm{a(Tx\#1)}}\right)} \\ &\quad \times \sqrt{\eta_{\mathrm{a(Tx\#2)}}\mathrm{Re}\left(Z_{\mathrm{a(Tx\#2)}}\right)} \\ &\quad \times B'_{10,10}\left(r_{0_{(\mathrm{Tx})}}, \theta_{0_{(\mathrm{Tx})}}, \phi_{0_{(\mathrm{Tx})}}, \theta_{1_{(\mathrm{Tx})}}, \phi_{1_{(\mathrm{TX})}}\right) \end{aligned} \tag{9}$$

$$\begin{aligned} Z_{12(\mathrm{Rx})} &= Z_{21(\mathrm{Rx})} \\ &= \sqrt{\eta_{\mathrm{a(Rx\#1)}}\mathrm{Re}\left(Z_{\mathrm{a(Rx\#1)}}\right)} \\ &\quad \times \sqrt{\eta_{\mathrm{a(Rx\#2)}}\mathrm{Re}\left(Z_{\mathrm{a(Rx\#2)}}\right)} \\ &\quad \times B'_{10,10}\left(r_{0_{(\mathrm{Rx})}}, \theta_{0_{(\mathrm{Rx})}}, \phi_{0_{(\mathrm{Rx})}}, \theta_{1_{(\mathrm{Rx})}}, \phi_{1_{(\mathrm{RX})}}\right) \end{aligned} \tag{10}$$

where $B'_{10,10}(r_0, \theta_0, \phi_0, \theta_1, \phi_1)$ is given again by (5).

If the antenna elements of the transmitting array are simultaneously matched, the realizable matching impedance can be given as in [10]

$$Z_{s1} = \frac{\alpha_1 + \sqrt{\Delta}}{2\mathrm{Re}\left(Z_{\mathrm{a(Tx\#2)}}\right)} \quad Z_{s2} = \frac{\alpha_2 + \sqrt{\Delta}}{2\mathrm{Re}\left(Z_{\mathrm{a(Tx\#1)}}\right)} \tag{11}$$

where

$$\begin{aligned} \alpha_1 = &-2j\mathrm{Re}\left(Z_{\mathrm{a(Tx\#2)}}\right)\mathrm{Im}\left(Z_{\mathrm{a(Tx\#1)}}\right) \\ &+ j\mathrm{Im}\left(Z_{12(\mathrm{Tx})}Z_{21(\mathrm{Tx})}\right) \end{aligned} \tag{12}$$

$$\begin{aligned} \alpha_2 = &-2j\mathrm{Re}\left(Z_{\mathrm{a(Tx\#1)}}\right)\mathrm{Im}\left(Z_{\mathrm{a(Tx\#2)}}\right) \\ &+ j\mathrm{Im}\left(Z_{12(\mathrm{Tx})}Z_{21(\mathrm{Tx})}\right) \end{aligned} \tag{13}$$

$$\begin{aligned} \Delta = &\left[2\mathrm{Re}\left(Z_{\mathrm{a(Tx\#1)}}\right)\mathrm{Re}\left(Z_{\mathrm{a(Tx\#2)}}\right) - \mathrm{Re}\left(Z_{12(\mathrm{Tx})}Z_{21(\mathrm{Tx})}\right)\right]^2 \\ &- \left|Z_{12(\mathrm{Tx})}Z_{21(\mathrm{Tx})}\right|^2. \end{aligned} \tag{14}$$

For the receiving array, the matching load impedances Z_{l1} and Z_{l2} can be calculated in the same manner.

When the mutual couplings at the transmitting and receiving arrays are considered, the channel matrix can be given as

$$\mathbf{H} = \mathbf{C_{RX}}\mathbf{H}_{\boldsymbol{\omega}}\mathbf{C_{TX}} \tag{15}$$

where $\mathbf{C_{RX}}$ is given in (8), $\mathbf{H}_{\boldsymbol{\omega}}$ is given in (6), and $\mathbf{C_{TX}}$ is given in (7) [8]. From (15), it can be shown that if the characteristics of the individual antenna and the geometrical parameters of the system are given, the channel matrix of the near-field 2 × 2 MIMO can be estimated by the proposed method.

If the channel matrix is given, the capacity of the MIMO can be estimated. According to the Shannon theorem, for the channel matrix $\mathbf{H}$, the capacity of a 2 × 2 MIMO is given as

$$C = \sum_{i=1}^{2} \log_2\left(1 + \lambda_i \frac{\gamma_0}{2}\right) = \log_2 \prod_{i=1}^{2}\left(1 + \lambda_i \frac{\gamma_0}{2}\right) \tag{16}$$

where λ_i is the ith eigenvalue of $\mathbf{H}^{\mathrm{H}}\mathbf{H}$, with the superscript$H$ representing the conjugate transpose, and γ_0 is the signal-to-noise ratio (SNR) [11]. In addition, the eigenvalues of $\mathbf{H}^{\mathrm{H}}\mathbf{H}$ can be calculated by the following

$$\det(\lambda\mathbf{I} - \mathbf{H}^H\mathbf{H}) = 0. \tag{17}$$

V. Example and Discussion

As an example, the characteristics of a 2 × 2 near-field MIMO, which is composed of an electric short dipole and a small 5-turn solenoidal loop, are investigated. As shown in Fig. 4, the length of the electric dipole is 5 cm and the radius of the solenoidal loop is 10 cm, and they are placed in a diagonal configuration ($\theta_{0(\mathrm{Tx})} = \theta_{0(\mathrm{Rx})} = 0.75\pi$, $\phi_{0(\mathrm{Tx})} = \phi_{0(\mathrm{Rx})} = 0.5\pi$) with a distance of 15 cm. In addition, the solenoidal loop of each antenna pair is $\pi/4$-rotated counterclockwise around the positive y-axis ($\theta_{1(\mathrm{Tx})} = \theta_{1(\mathrm{Rx})} = 0.25\pi$, $\sigma_{1(\mathrm{Tx})} = \sigma_{1(\mathrm{Rx})} = 0$). The operating frequency is set to 13.56 MHz, and the impedance characteristics Z_{a} of each antenna are analyzed by the commercial method of moments (MoM)-based electromagnetic (EM) simulator.

When the distance between the antenna pairs is set in the parallel and diagonal configurations as shown in Fig. 4, the capacity

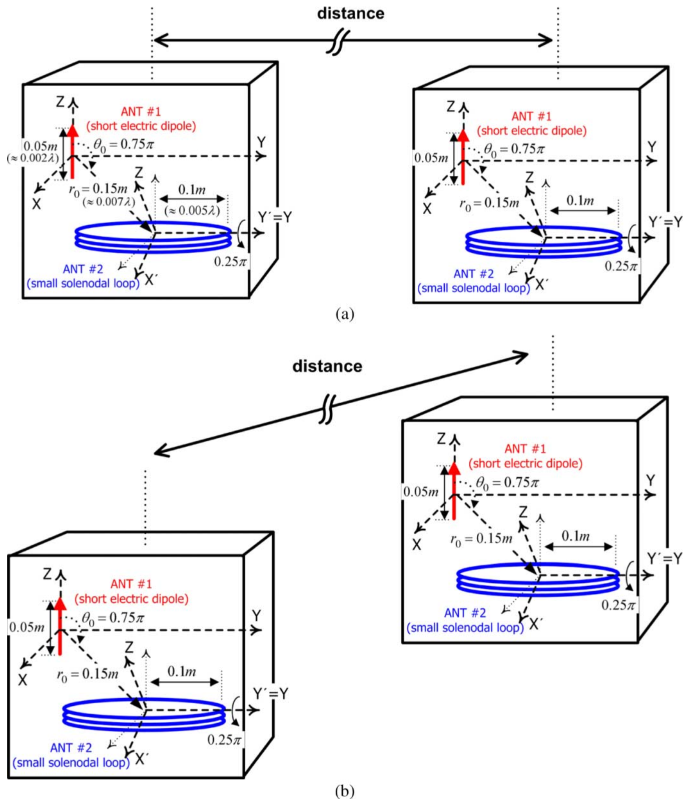

Fig. 4. Example of a near-field 2 × 2 MIMO. (a) In a parallel configuration. (b) In a diagonal configuration.

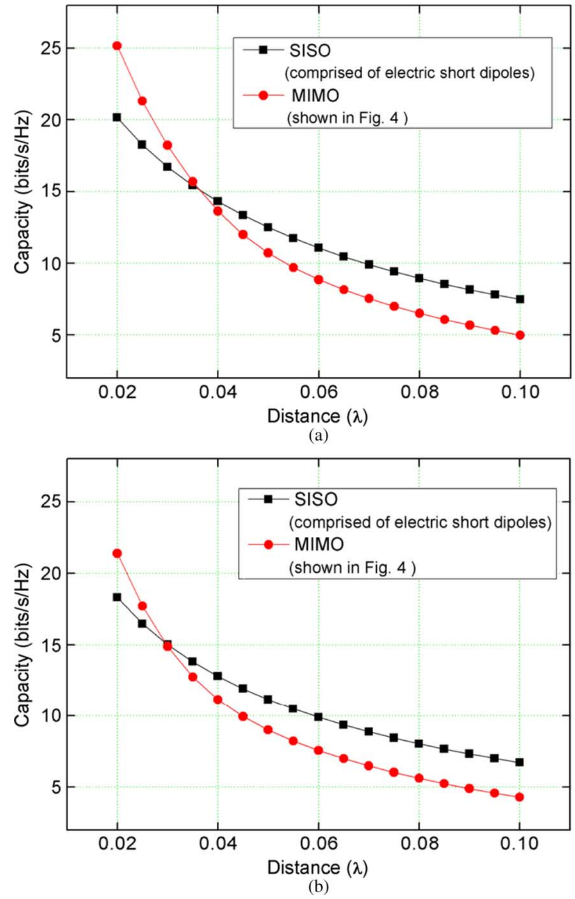

Fig. 5. Capacity calculation results of a near-field 2 × 2 MIMO under the condition of the equal power allocation with the same amount of the total transmitted power ($\gamma_0 = 30$ dB). (a) In a parallel configuration. (b) In a diagonal configuration.

characteristics of the near-field 2 × 2 MIMO, where the transmitting SNR is given as 30 dB under the equal power allocation condition with the same amount of total transmitted power, are calculated by the proposed method. The calculated results for the different conditions are compared in Fig. 5.

According to the results, it can be shown that the 2×2 near-field MIMO using an electric short dipole and a small solenoidal loop can increase the channel capacity. The capacity performance of the given example, however, appears degraded in the region of a distance larger than 0.03λ compared to the single-input–single-output (SISO), which uses only short electric dipoles as transmitting and receiving antennas. This result is due to the fact that, in this example, the efficiency of the small solenoidal loop is much smaller than the short electric dipole.

VI. Conclusion

By using an antenna array composed of a short electric dipole and a small solenoidal loop, a near-field 2 × 2 MIMO can be constructed because the orthogonal TM_{10} or TE_{10} spherical mode can serve as an individual channel. In this case, the transmission characteristics of the near-field MIMO can be estimated by the mode-based method because the interaction of the small antennas can be simply derived based on the addition theorem of the spherical waves. Mutual coupling at the transmitting and receiving arrays can also be calculated in a similar manner. Hence, using the proposed method, channel characteristics can be determined for a near-field MIMO where a mutual coupling between antenna elements exists, and the capacity of the MIMO can also be effectively estimated.

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