# Tapered Unit Cell Control of a Sinusoidally Modulated Reactance Surface Antenna

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***Abstract*—A design procedure for synthesizing the desired beam pattern of the sinusoidally modulated reactance surface (SMRS) antenna is proposed. This methodology is a type of aperture tapering technique, where cosine distribution of aperture illumination was applied to the SMRS antenna. The unit cell of the SMRS antenna allows control of the leakage constant while keeping phase constant almost constant value in some allowable surface impedance modulation range; this characteristic provides the aperture amplitude tapering technique to the SMRS antennas. To verify this proposed concept, a prototype SMRS antenna composed of nine tapered unit cells was implemented using a printed circuit board process. The designed antenna radiates at 35° off broadside at 10 GHz. We obtained 16.6 dBi of measured antenna gain and the normalized sidelobe level was about −14.33 dB.**

***Index Terms*—Aperture tapering, leaky-wave antennas, sidelobe level, sinusoidally modulated reactance surface antenna (SMRS).**

## I. Introduction

FOLLOWING initial research on the guidance and radiation properties of sinusoidally modulated reactance surfaces (SMRS) [1], studies were conducted with regard to SMRS antennas [2]–[6]. The SMRS antennas have attracted a great deal of interests due to their low profile, light weight, and highly directive beam pattern characteristics. Based on these advantages, researches have been undertaken on SMRS antennas for space applications [4]–[6].

In terms of the radiation pattern of SMRS antennas, uniform modulation of a reactance surface was attempted to verify the general radiation characteristic of the SMRS antenna [2]. This uniform modulation maintains the same leakage constant of the radiated field along the antenna and leads to the existence of high level of sidelobe near the main beam. In [4]–[6], desired SMRS antenna patterns were synthesized using calculated design curves [4], [5] or by following iterative processes to determine detailed antenna characteristics such as amplitude, phase, and polarization [6]. Also, appropriate control of the design parameters led to the independent control of the leakage constant and phase constant of the radiated field and near field focusing was achieved [7].

There are a number of commonly used techniques for the synthesis of desired radiation patterns for leaky-wave antennas [8], [9]. For the continuous leaky wave antenna [10], the geometry of the antenna is adjusted continuously in order to obtain the desired radiation field characteristics. In the case of quasi-uniform leaky-wave antennas whose unit cell size (period) is small compared to the guided wavelength, there was an array factor approach [11] that can easily analyze and synthesize the desired radiation pattern of tapered leaky-wave antennas.

In this letter, we applied an aperture amplitude tapering technique to an SMRS antenna. Cosine distribution of the aperture illumination was achieved with nine tapered SMRS antenna unit cells by controlling the modulation factor of each unit cell. Both the simulated and measured results lowered the sidelobe level of the radiation pattern when compared with the uniformly modulated case, validating the design procedure. The details of the design procedure and experimental results are presented and discussed in the following sections.

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## II. Proposed Antenna Design

### A. Theoretical Concept and Unit Cell of the SMRS Antenna

In this section, we briefly introduce the theoretical concepts and unit cell model for the SMRS antenna. Detailed theories and the basic principles of SMRS antennas are well organized in [1] and [2].

An SMRS is one whose surface impedance value has a sinusoidal form along the longitudinal direction. The surface impedance is defined as the ratio of the tangential electric field to the tangential magnetic field of the guided surface wave. Therefore, the mathematical expression for sinusoidally modulated reactance along the direction of propagation is as follows:

$$\eta_{\mathrm{surf}}(z) = j\eta_0 X' \left[1 + M\cos\left(\frac{2\pi z}{a}\right)\right]. \tag{1}$$

The above-mentioned equation can be plotted as in Fig. 1, where $X$ is the average surface reactance, $M$ is the modulation factor, $a$ is the period of the sinusoidal function, and $X'$ is the average surface reactance divided by the free-space wave impedance $(X' = X/\eta_0)$. gMIN and gMAX in Fig. 1, refers to the gap sizes for the implementation of the SMRS according to the reactance values, are going to be discussed later. When one of the Floquet mode generated from the periodicity of the SMRS becomes a fast wave $(\beta_n < k_0)$, a radiating leaky-wave occurs and this characteristic can be utilized as a leaky-wave antenna [8].

For the implementation of such an SMRS, an array of conductor strips over a ground plane is widely used. The modulation of surface reactance is achieved by varying the gap sizes between the strips. Fig. 2 provides an illustration of an SMRS antenna unit cell. Sinusoidally modulated reactance values for the unit

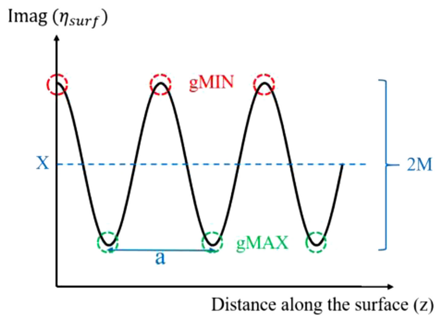


Fig. 1. Sinusoidal form of surface reactance and relative gap sizes at maximum and minimum reactance for physical realization of the SMRS.

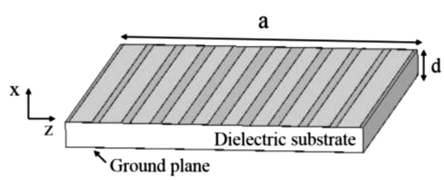


Fig. 2. SMRS antenna unit cell composed of ten metallic strips on a dielectric substrate over ground plane.

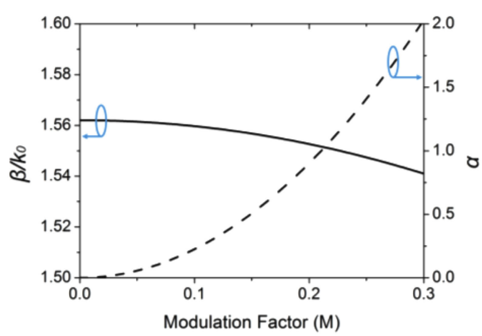


Fig. 3. Variation of normalized phase constant and leakage constant according to the modulation factor *M* when $X'$ is 1.2 and $a = 30$ mm at 10 GHz.

cell are sampled at ten points within a period; this is physically realized using strips and gaps between them. The one-tenth part of the unit cell, which is composed of two conductor strips and a gap is going to be called as a unit segment. The variation of surface reactance of a unit segment according to the gap sizes can be calculated applying transverse resonance condition to the equivalent transmission line model of the unit segment the SMRS antenna [2].

In the reactance profile of SMRS as shown in Fig. 1, the maximum reactance point corresponds to the minimum value of the gap (gMIN) for unit segment. As the area composed of conductor within a unit segment increases, the reactive coupling become more intense so the reactance increases. The maximum value of the gap (gMAX) gives the minimum surface reactance as an antipode of gMIN case.

### B. Tapered SMRS Antenna Design

Mathematical manipulation and approximation lead to the derivation of the dispersive relation of guided waves on the SMRS, as shown in [2, eq. (2)]. Based on this equation, we can plot the variations of the phase constant ($\beta$) and the leakage constant ($\alpha$) according to the modulation factor (*M*) at 10 GHz as shown in Fig. 3. In this case, $X'$ is 1.2 and the length of the unit cell($a$) is 30 mm. As can be inferred from the graph, the variation of $\beta$ compared to $\alpha$ is nearly constant and this characteristic allows tapering of the $\alpha$ of each unit cell while keeping $\beta$ almost uniform for all unit cells along the antenna. This characteristic is maintained for an allowable range when the modulation factor (*M*) has a small value (<0.6).

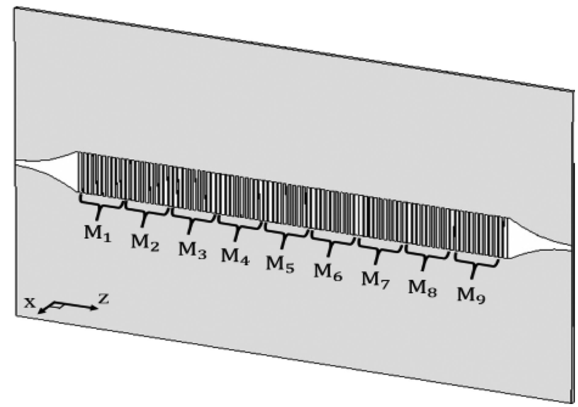


Fig. 4. Proposed 3-D model of a tapered SMRS antenna marked with nine discrete modulation factors.

We applied an aperture amplitude tapering technique to the SMRS antenna utilizing the above characteristic of the unit cell. A cosine distribution of the aperture illumination was achieved with nine discrete SMRS unit cells in order to obtain a sharp main beam while keeping a low sidelobe level. Fig. 4 shows the configuration of our proposed tapered SMRS antenna whose nine unit cells have different modulation factors according to the desired aperture illumination distribution. The calculation of the leakage constants for each unit cell provides the determination of modulation factors for the unit cells.

The leakage constants of unit cells can be determined by equating the aperture illumination function $\tilde{M}(\mathrm{z})$ to the normalized sine function

$$\tilde{M}(z) = \sqrt{\alpha(z)}e^{-\int_0^z \alpha(\tau)d\tau}e^{-j\int_0^z \beta(\tau)d\tau}. \tag{2}$$

In our case, the leakage constant is assumed to be piecewise constant within single unit cell and only the real part of the exponent contributes to the amplitude of the aperture illumination. Therefore, (2) is treated in a simpler form and equated with the normalized sine function

$$\sin(\pi z/L) = \sqrt{\alpha(z)}e^{-\int_0^z \alpha(\tau)d\tau} \tag{3}$$

where $L$ is the length of the radiating part of the SMRS antenna and in this case, $L = 9 \times a$.

Some manipulation of (3) gives the analytic expression of $\alpha(z)$

$$\alpha(z) = -0.5 \times \left(\frac{0.5 - 0.5\cos(\pi z/L)}{0.5z - L/4\pi \sin(2\pi z/L) - 0.5}\right). \tag{4}$$

With the equation obtained for $\alpha$, we calculated the values of $\alpha$ for each unit cell by putting the sine function values at the midpoints of each unit cell. The calculated leakage constants for each unit cell and the corresponding required surface reactance modulation factors (*M*) are listed in Table I. To examine the variation of phase constant according to modulation factors, extracted $\beta/k_0$ values are verified and required gMIN and gMAX (in mm) for the implementation with the Rogers RT/Duroid 6006 with 2.54 mm of thickness for each unit cell are also tabulated in Table I. From Fig. 3 and the values in Table I, it can

TABLE I
CALCULATED LEAKAGE CONSTANTS ($\alpha$), CORRESPONDING MODULATION FACTORS ($M$), PHASE CONSTANTS ($\beta$) ACCORDING TO FIG. 3 AND gMIN AND gMAX FOR EACH UNIT CELL

| Cell # | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | 0.03 | 0.25 | 0.6 | 1.1 | 1.5 |
| M | 0.036 | 0.105 | 0.163 | 0.22 | 0.258 |
| $\beta/k_0$ | 1.5617 | 1.5594 | 1.5557 | 1.5506 | 1.5465 |
| gMIN | 0.9097 | 0.7616 | 0.6494 | 0.5734 | 0.5393 |
| gMAX | 1.0836 | 1.2643 | 1.4285 | 1.5574 | 1.6211 |

| Cell # | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| $\alpha$ | 1.3 | 0.77 | 0.3 | 0.04 |
| M | 0.235 | 0.185 | 0.115 | 0.042 |
| $\beta/k_0$ | 1.5489 | 1.554 | 1.5589 | 1.5616 |
| gMIN | 0.5511 | 0.611 | 0.7271 | 0.8973 |
| gMAX | 1.5985 | 1.4916 | 1.3119 | 1.0975 |

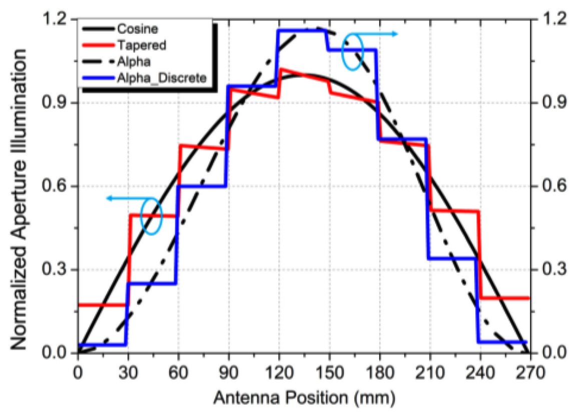


Fig. 5. Plot of the cosine distribution function, the tapered aperture illumination function, ideal distribution of alpha, and discrete alpha values along the antenna position.

be seen that the required modulation factors for cosine aperture illumination are within small value range (<0.6) and maximum radiating beam angle deviation was calculated to be 1.5°, keeping almost same angle while maintaining the desired aperture illumination along the antenna.

With the calculated leakage constants of the nine unit cells, we plotted the amplitude of a tapered aperture illumination case in Fig. 5 and compared it with the cosine distribution. The desired continuous leakage constant distribution from (4) and discrete alpha values for each unit cells are also shown. The amplitude of the tapered case follows a cosine-like distribution composed of nine discretized exponentially decaying functions validating the calculation of leakage constants using the proposed procedure. Based on the leakage constants and the unit cell length, we calculated the efficiency of our proposed antenna with the method introduced in [8] for general leaky wave antennas and 30% of efficiency was obtained. This efficiency can be increased with the longer antenna length and more careful design approach on the tapering method.

The gap size for each segment of the designed tapered SMRS antenna is shown in Fig. 6. According to the tapered modulation factor ($M$) values as shown in Table I, the difference of the gMIN and gMAX is the largest in the middle of the antenna and becomes smaller as it goes to the edges of the antenna. This modulated profile of the gap sizes led to the amplitude tapering of the aperture field of the SMRS antenna in contrast to

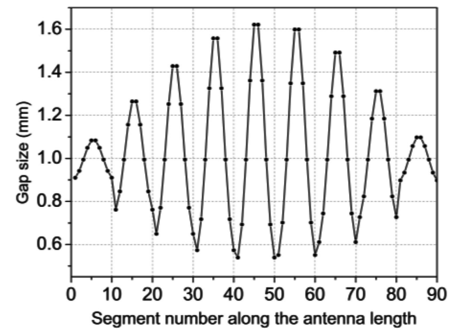


Fig. 6. Plot of the gap size of the designed tapered SMRS antenna along the antenna length direction.

the uniform modulation of Fig. 1, which provides exponential amplitude distribution of aperture fields and higher sidelobe level.

The radiation pattern for the proposed antenna can be obtained calculating the Fourier transform of the aperture illumination [9]

$$R(\theta) = \int_0^L \tilde{M}_n(z) e^{jk_0 z \sin\theta} dz \tag{5}$$

in our case, the aperture illumination functions are different for each unit cell and can be represented as follows.

$$\tilde{M}_n(z) = \sqrt{\alpha_n} e^{-\alpha_n z} e^{-j\beta_n z}, \ n = 1, 2 \ldots 9 \tag{6}$$

where the values for $\alpha$ and $\beta$ for unit cell number are listed in Table I. We calculated the radiation pattern of the proposed tapered SMRS antenna based on (5) and the result will be presented and discussed in Section III.

## III. SIMULATED AND EXPERIMENTAL RESULTS FOR THE PROPOSED TAPERED SMRS ANTENNA

The proposed tapered SMRS antenna was simulated and implemented on the Rogers RT/Duroid 6006 substrate with a thickness of 2.54 mm. Electromagnetic (EM) simulation of the proposed SMRS antenna was performed using commercial EM tool Computer Simulation Technology (CST) and its three-dimensional (3-D) model is shown in Fig. 4. The proposed antenna is composed of nine unit cells whose lengths ($a$) are 30 mm and impedance transformer sections for the 50Ω line are added at both ends of the antenna. The antenna is designed with a center frequency of 10 GHz with the radiation angle of 35°. The return loss and insertion loss of the proposed antenna were simulated and measured with a 50Ω termination load at the end of the antenna. The simulated and measured results from 9 to 11 GHz are shown in Fig. 7 and the two results are in a good agreement. The radiation patterns of the proposed antenna were also measured. The measurement setup for the fabricated antenna in an anechoic chamber is shown in Fig. 8. The measured pattern at 10 GHz is compared with the simulated result and calculated pattern from (5) is also plotted in Fig. 9 all the results are normalized with their maximum values. The simulated and measured results show good agreement and the sidelobe level of the measured result was −14.33 dB at −25° and this is the radiation from the −2nd Floquet mode.

Although the calculated result using (5) is slightly different from the simulated and measured results, the calculated result suggests a feasible value for the maximum sidelobe level when compared with the simulated and measured results and estimates

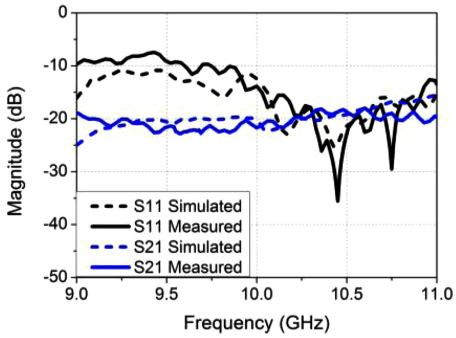


Fig. 7. Simulated and measured results of return loss and insertion loss of the proposed antenna.

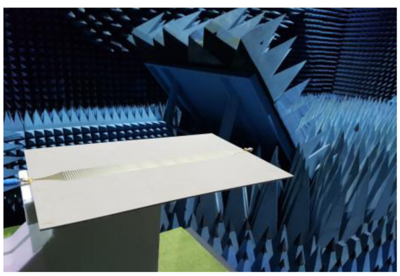

Fig. 8. Radiation pattern measurement setup for the proposed antenna in an anechoic chamber.

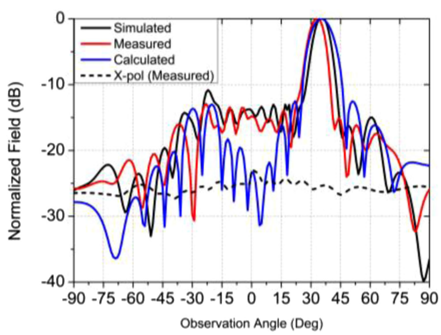


Fig. 9. Simulated, measured, calculated radiation patterns and measured cross polarization at 10 GHz.

the main beam direction at 35°, including radiation from the $-2$nd Floquet mode around $-25°$.

The simulated gain of the proposed antenna was 17.56 dBi and the measured gain was 16.6 dBi at 10 GHz. The measured cross polarization was below $-23$ dB over all directions.

Fig. 10 shows the simulated and measured radiation patterns of the proposed SMRS antenna at 9.5 and 10.5 GHz, respectively. The simulated and measured results are in a good agreement for both frequency cases and the average beam squint was 16.5°/GHz.

To validate the design procedure for our proposed antenna, we compared the measured radiation pattern of our tapered SMRS antenna with the simulated radiation pattern of a uniformly modulated case ($M = 0.2$) in Fig. 11. Both patterns are normalized with the maximum gain value of uniformly modulated case (18.1 dBi). As can be observed from the results, the highest sidelobe for the uniformly modulated case was observed at 45° with $-8$ dB level and this was suppressed in the tapered case at the expense of increase of other sidelobe level away from the main beam direction achieving a quite uniform

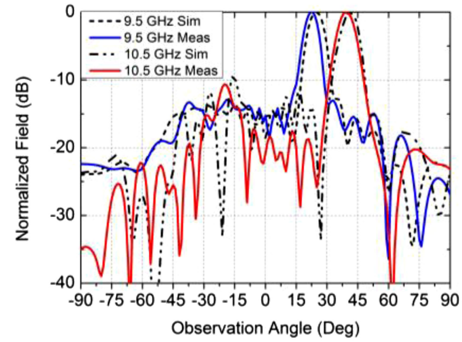


Fig. 10. Simulated and measured radiation patterns at 9.5 and 10.5 GHz.

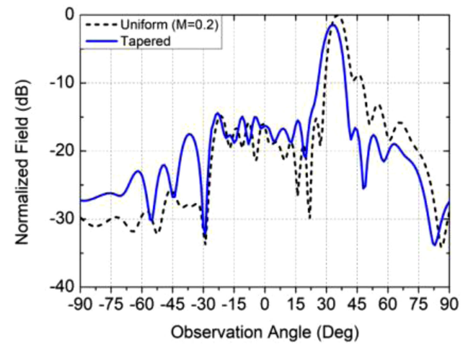


Fig. 11. Comparison of the radiation patterns for the uniformly modulated (simulated, $M = 0.2$) and proposed tapered SMRS antenna (measured) at 10 GHz.

sidelobe level. About 1.5 dB of maximum gain drop was observed for proposed tapered case because the amplitude tapering for the sidelobe suppression has reduced the aperture efficiency of the proposed antenna.

## IV. Conculsion

In this letter, we proposed an SMRS antenna whose aperture illumination has a cell by cell tapered amplitude distribution. The tapered amplitude distribution for the SMRS antenna was achieved based on the characteristic of the unit cell, which allows control of the leakage constant $(\alpha)$ according to the surface reactance modulation factor ($M$), while keeping the phase constant $(\beta)$ almost constant. The design procedure for the desired aperture amplitude illumination was demonstrated. For the validation of the proposed concept, an SMRS antenna composed of nine unit cells having cosine aperture illumination distribution was simulated and measured. The measured radiation pattern of the proposed antenna showed a uniform sidelobe level with a normalized value of $-14.33$ dB and 16.6 dBi of maximum gain was obtained at the designed frequency of 10 GHz. The results validate the proposed concept when compared to the uniformly modulated case and implies that not only cosine distribution but also other aperture amplitude distributions with uniform phase constants can be applied to SMRS antennas. Also, beyond our proposed concept, more delicate and precise design could be achieved using other parameters, such as period $a$ and average surface reactance $X$ in addition to the local gap $g$ in order to control both the amplitude and the phase distribution. This will provide more flexibility on the synthesis of the radiation patterns both in near- and far-field regions as demonstrated in [7], including further improvement of the sidelobe level than our result.

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