

# AN ADVANCED EQUIVALENT CIRCUIT OF SPIRAL-SHAPED DEFECTED GROUND STRUCTURE

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**Abstract** — An advanced modeling technique to achieve the equivalent circuit of spiral-shaped defected ground structures (DGS) is presented. The equivalent circuit is composed of a short-circuited stub and an inductor to express the spurious frequency as well as the resonant frequency. The elements of the equivalent circuit are extracted from the related theories on step impedance resonators (SIR) and the S-parameters calculated by electromagnetic simulation. While the existing equivalent circuit model adopts the SIR composed of transmission line elements with the same-length[1], the proposed equivalent circuit uses the SIR composed of different length of transmission line elements. It is shown that the modeled S-parameters from the proposed equivalent circuit are in excellent agreement with those obtained from the electromagnetic simulation.

**Index Terms** — periodic structures, defected ground structures, DGS, spiral-shaped DGS

## I. INTRODUCTION

It is well known that planar transmission lines combined by periodic structures such as photonic bandgap (PBG) and defected ground structures (DGS) have larger slow-wave factor (SWF) than standard transmission lines [2-4]. In addition, spiral-shaped DGS patterns for transmission lines have been proposed in [1] and [5]. It is very interesting that “the transmission line with spiral-shaped DGS” (“Spiral-DGS line”) has higher SWF than the transmission lines with dumbbell-shape DGS as well as standard transmission lines. This means that a great potential application of Spiral-DGS is expected in the design of high frequency circuits.

It is essential to model the exact equivalent circuit of Spiral-DGS to understand the frequency responses and to expand the usages. The equivalent circuit of Spiral-DGS has been proposed previously using the simple step impedance resonator (SIR) [1].

In this work, based on the previous work and more general theory of step impedance resonator, an advanced equivalent circuit model of Spiral-DGS is

proposed. The lengths of three transmission line elements in the SIR are not fixed to be the same, but differ from each other. There are a lot of degree of freedom in the SIR, and this produces much more exact equivalent circuit than the existed method.

## II. STRUCTURE OF SPIRAL DGS

Fig. 1 (a) shows the structure of a microstrip line combined by a Spiral-DGS on the ground plane, as an example. The dimensions of Spiral-DGS are designated as A, B, W1, W2, SW, and L. Here, the length of the connecting slot is fixed to be L, which corresponds to the width of 50Ω microstrip line (W50), for the convenience. L may differ from W50.

Fig. 1 (b) presents the electromagnetically (EM) calculated S-parameters for Fig. 1(a). The dielectric constant ( $\epsilon_r$ ) and thickness(H) of the selected substrate are 2.6 and 0.5mm, respectively. The dimensions of Spiral-DGS are; A=2mm, B=3.4mm, W1=W2=SW=0.2mm, L=1.4mm=W50.

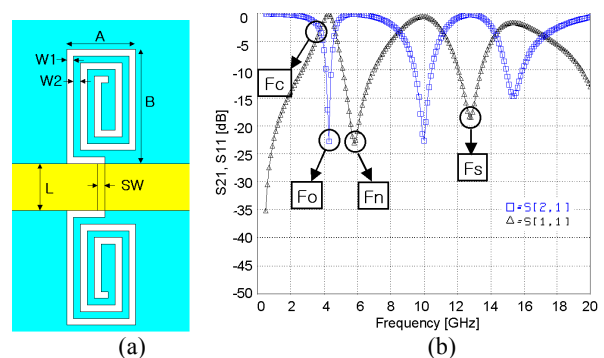


Fig. 1 (a) Unit Spiral-DGS on the ground plane of microstrip line ( $\epsilon_r=2.6$ , A=2mm, B=3.4mm, W1=W2=SW=0.2mm, L=1.4mm, Substrate thickness=20mils) (b) Calculated S-parameters by EM simulation

It is shown that resonant and notch frequencies appear repeatedly. This proves intuitively that the equivalent circuit of Spiral-DGS should contain at least one distributed element rather than lumped elements only.

There are four meaningful frequencies,  $F_o$ ,  $F_n$ ,  $F_s$ , and  $F_c$  in Fig. 1(b). They are called conveniently in this work as the first resonant frequency, the first notch frequency, spurious frequency, and 3dB cutoff frequency, respectively.

### III. THE PREVIOUS EQUIVALENT CIRCUIT [1]

Fig. 2 shows the equivalent circuit of Spiral-DGS. There is a short-circuited stub having SIR, equivalently added inductance ( $L_s$ ), and resistance ( $R$ ).  $Z_o$  and  $\phi$  represent the port impedance and length of the port feeding line.

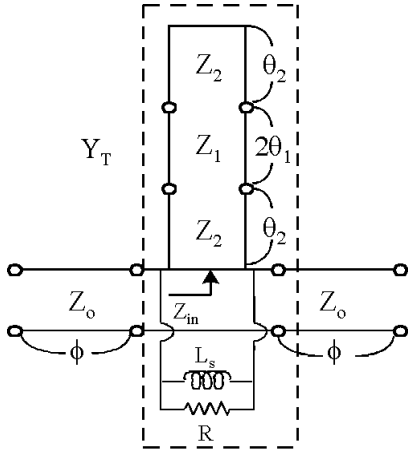


Fig. 2 Equivalent circuit using a short-circuited stub having SIR

In the previous equivalent circuit, the length of transmission line elements is fixed to be the same, i.e.  $\theta_1 = \theta_2 = \theta$ , in order to simplify the short-circuited stub. The input impedance and admittance are expressed as eqs. (1) ~ (5). Here,  $K$  is  $Z_2/Z_1$  and positive because  $Z_1$  and  $Z_2$  are finite positive values.

$$Z_{in} = jZ_2 \frac{2(1+K)(1-K \cdot \tan^2 \theta) \tan \theta}{K - 2(1+K+K^2) \tan^2 \theta + K \cdot \tan^4 \theta} = jZ_2 \cdot F(\omega) \quad (1)$$

$$F(\omega) = \frac{2(1+K)(1-K \cdot \tan^2 \theta) \tan \theta}{K - 2(1+K+K^2) \tan^2 \theta + K \cdot \tan^4 \theta} \quad (2)$$

$$Y_{in} = -jY_2 \frac{1}{F(\omega)} \quad (3)$$

$$Y_T = Y_{in} + \frac{1}{R} - j \frac{1}{\omega L_s} = G + jB_T(\omega) \quad (4)$$

$$B_T = -\frac{Y_2}{F(\omega)} - \frac{1}{\omega L_s} \quad (5)$$

Because  $Z_{in}=0$  at  $F_n$ , the numerator of (1) is zero, and eq. (6) is obtained. In addition, because the denominator may be  $\infty$  in order that  $Z_{in}=0$ , eq. (7) is obtained at  $F_s$ . Then,  $\theta_s = \pi/2$ . Now,  $K$  is calculated from eq. (8) and (9).

$$\theta_n = \tan^{-1} \frac{1}{\sqrt{K}} \quad (6)$$

$$\tan \theta_s = \infty \quad (7)$$

$$\frac{\theta_s}{\theta_n} = \frac{f_s}{f_n} = \frac{\pi/2}{\tan^{-1}(1/\sqrt{K})} \quad (8)$$

$$K = \frac{1}{\left( \tan \frac{\pi f_n}{2 f_s} \right)^2} \quad (9)$$

At  $F_o$ ,  $B_T$  must be 0, which produces eq. (10). Also,  $B_T$  at  $F_c$  should be equal to the susceptance value at the normalized cutoff frequency of 1-pole prototype low pass filter. This relation generates eq. (11), where  $g_k$  and  $Z_o$  are the normalized element value and port impedance level, respectively.  $Z_2$  and  $L_s$  are calculated by equating eq. (10) and (11). Finally  $Z_1$  is obtained from  $K$  and  $Z_2$ . The resistance ( $R$ ) is calculated easily from the S-parameters at resonance.

$$-\frac{Y_2}{F(\omega_o)} - \frac{1}{\omega_o L_s} = 0 \quad (10)$$

$$-\frac{Y_2}{F(\omega_c)} - \frac{1}{\omega_c L_s} = \frac{1}{g_k Z_o} \quad (11)$$

The obtained element values are;  $Z_1=35.76\Omega$ ,  $Z_2=48.06\Omega$ ,  $L_s=2.0007\text{nH}$ ,  $R=1,313\Omega$ . Fig. 3 shows the characteristics of the extracted equivalent circuit.

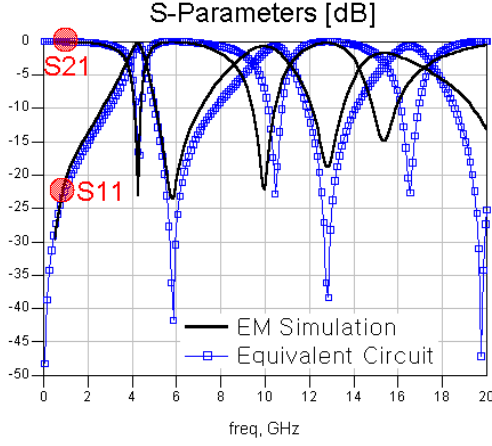


Fig. 3 Characteristics of the equivalent circuit with Fig. 1(b) overlapped.

#### IV. THE ADVANCED EQUIVALENT CIRCUIT

However, discrepancies are observed at the second and third resonant frequencies in Fig. 3. This is caused by the simplification of SIR, i.e. by fixing  $\theta_1 = \theta_2 = \theta = \pi/2$  (at  $F_s$ ). However, in practice, there exist a great number of combinations of  $\theta_1$  and  $\theta_2$  according to  $K$  [6].

If  $\theta_1 \neq \theta_2$ , eq. (1) does not hold any longer, and the input impedance of SIR can be expressed as eq. (12). When the total length of SIR is  $\theta_T$ ,  $\theta_2$  is equal to  $\theta_T/2 - \theta_1$ . Eq. (13) can be obtained using the fundamental notch condition ( $1 = K \tan \theta_1 \tan \theta_2$ ). Fig. 4 depicts the lots of possible combination of  $\theta_1$  and  $\theta_T$  when  $K$  is the variable parameter. If  $K=1$  for the simple case,  $Z_1 = Z_2$  and  $\theta_1 = \pi$ .

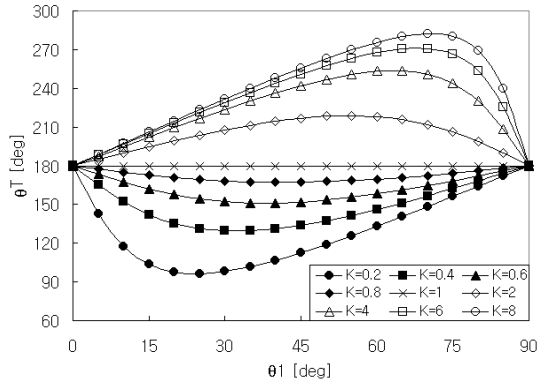


Fig. 4 Relation between  $\theta_1$  and  $\theta_T$  with  $K$  as parameter

$$Z_{in} = jZ_2 \frac{2(\tan \theta_1 + K \tan \theta_2)(1 - K \tan \theta_1 \tan \theta_2)}{K(1 - \tan^2 \theta_1)(1 - \tan^2 \theta_2) - 2(1 + K^2) \tan \theta_1 \tan \theta_2} \quad (12)$$

$$\tan \frac{\theta_T}{2} = \frac{1}{1 - K} \left( \frac{K}{\tan \theta_1} + \tan \theta_1 \right) \quad (13)$$

There is another condition, that is,  $\tan \theta_1 = -K \tan \theta_2$ . Because  $K$  is positive,  $\tan \theta_1$  and  $\tan \theta_2$  should have the opposite sign. This restricts the possible combination of  $\theta_1$  and  $\theta_2$ . Even though a great number of combination can be thought as shown in Fig. 4, only the  $(\theta_1, \theta_2)$  pairs which satisfy  $\tan \theta_1 = -K \tan \theta_2$  can be chosen. Finally the possible pairs are illustrated in Fig. 5 for various  $K$ .

Once  $K$  and the pair of  $\theta_1$  and  $\theta_2$  are determined, the remaining procedures to get  $Z_1$ ,  $Z_2$ , and  $L_s$  are the same as above. As an example, when  $K$  and  $\theta_1$  are chosen to be 1.6 and  $30^\circ$ , respectively, the calculated  $Z_1$ ,  $Z_2$ , and  $L_s$  are  $37.9 \Omega$ ,  $60.7 \Omega$ , and  $2.1 \text{ nH}$ , respectively. Fig. 6 shows the S-parameters of the newly extracted equivalent circuit using the proposed method. Much better agreement than Fig. 3 is achieved. Fig. 6 proves that the exactitude of the proposed equivalent circuit has been obtained successfully..

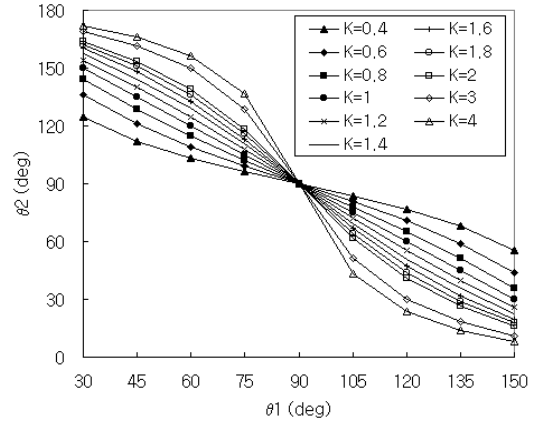


Fig. 5 Combination of  $\theta_1$  and  $\theta_2$  with  $K$  as parameter

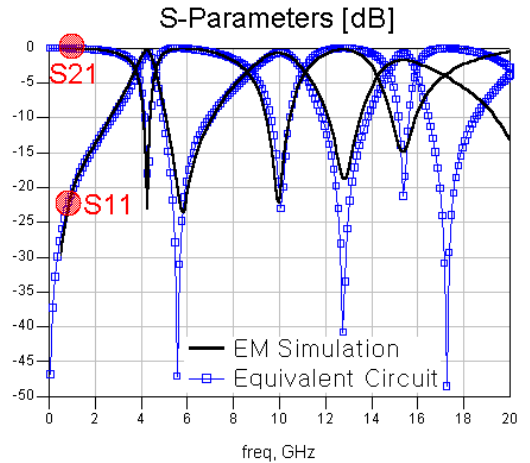


Fig. 6 Characteristics of the modified equivalent circuit with Fig. 1(b) overlapped.

## V. CONCLUSION

The previous equivalent circuit for Spiral-DGS has been reviewed, and the new method to extract the improved equivalent circuit of has been proposed. Using the related equations of SIR, the lots of possible combinations of the length of transmission line element in SIR have been calculated.

Depending on the proposed method, a Spiral-DGS under the microstrip line has been model, and the electrical characteristics of the equivalent circuit were calculated on Agilent Advanced Design System (ADS), a circuit simulator. The S-parameters of the equivalent circuit excellently agree with the EM simulation up to the third resonant frequency.

The proposed method can be applied to the Spiral-DGS for CPW as well as microstrip line. Because exact equivalent circuit is available, it is expected that the application of Spiral-DGS will be expanded to the design of various high frequency circuits.

## ACKNOWLEDGEMENT

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