THE METHOD OF AUXILIARY SOURCES FOR TWO DIMENSIONAL TRANSIENT SCATTERING ANALYSIS

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Abstract: The Method of Auxiliary Sources (MAS) has been used successfully for the numerical solution of a variety of electromagnetic problems, especially in frequency domain. There have been little efforts to apply MAS for time domain analysis. And the published articles take the explicit scheme for the marching-on-in-time (MOT) procedure, so the results suffer from the late-time instability. In this paper, we present the implicit scheme for the stable MOT procedure.

1. INTRODUCTION

The Method of Auxiliary Sources (MAS) is an efficient numerical technique for the solution of the boundary value problems arising in electromagnetic analysis [1]. MAS was introduced, named, and developed by a research group in Georgia (part of the former Soviet Union) [2]. Diverse research groups have independently developed very similar methods under different names [3]-[5]. Reference [1] introduces various aspects of MAS including historical perspectives, fundamentals of the method, and current status of the research activities, hence interested readers are referred to that article.

To explain the basic idea behind MAS, a simple two dimensional scattering problem in frequency domain is presented first. Solution procedures of MAS are demonstrated against that of the standard surface integral equation technique (SIE).

Fig. 1a shows the problem geometry. There is a perfect electrically conducting (PEC) scatterer and it is illuminated by an incident plane wave. In the standard SIE, the unknown currents (chosen current bases with unknown coefficients) are distributed on the scatterer surface. Then these unknown coefficients are solved for using the Method of Moments (MoM) [6]. In contrast to this, when applying MAS, discrete Auxiliary Sources (ASs) are located on the auxiliary surface enclosed by the physical scatterer surface, as shown in Fig. 1b. This auxiliary surface is usually conformal to the physical scatterer surface, but not necessarily is. Then the problem is solved imposing the boundary condition that the tangential electric field vanishes at the physical scatterer surface in the same way as the standard SIE. When the solution is completed, the coefficients of ASs are obtained, and then the needed quantities such as the surface currents and the radar cross section (RCS) can be calculated using these values. The coefficients of ASs themselves do not have physical significance.

In summary, MAS adopos the discrete sources

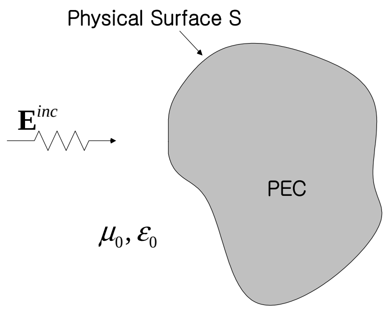

Fig. 1a. A PEC scatterer illuminated by an incident wave.

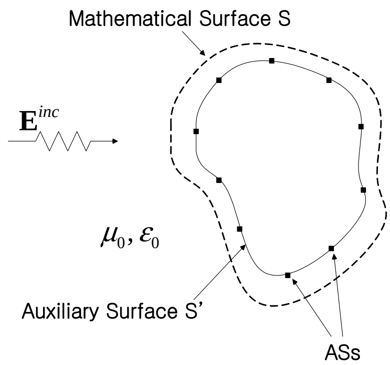

Fig. 1b. MAS model equivalent to the situation in Fig. 1a.

located in the inside of the physical scatterer and distant from the surface. Therefore the numerical integrations and singularity extractions are not needed, hence the numerical procedure is more simple and efficient than that of the standard SIE. All the advantages of MAS in frequency domain are directly applicable to MAS in time domain.

2. MAS IN TIME DOMAIN

Before we start the discussion of MAS in time domain, brief review of the Time Domain Integral Equation (TDIE) method seems to be in order. In recent years, TDIE for scattering analysis has received much attention and is believed to be in a mature stage, although a few problems concerning stability and accuracy still remain. The most widely used scheme is to discretize the scatterer with triangular patches and perform Marching-on-in-Time (MOT) with Rao-Wilton-Glisson (RWG) basis function in space and linear interpolation in time. In this scheme, basis functions for space and time are decoupled to avoid time-consuming space-time integrations and the retarded time is accounted for only approximately.

Since MAS adopts the discrete impulsive ASs, space-time integrations are not needed and the retarded time can be accounted for accurately. This is the advantage of MAS in time domain over the standard SIE based TDIE.

But, there have been little efforts to apply MAS in time domain and the published articles take the explicit scheme for the MOT procedure [7]. As is widely known, the explicit MOT scheme suffers from the late-time instability. In this paper, we present the implicit scheme for the MOT procedure.

3. FORMULATION

The governing equation is given by the boundary condition that the tangential electric fields vanish at the PEC surface.

$$\mathbf{E}^{inc}(x,y,t)+\mathbf{E}^{scat}(x,y,t)\Big|_{\tan}=0 \qquad (1)$$

For two dimensional TM to z case, the electric fields have z component only and the scattered field can be represented by a summation of contributions from the individual ASs as

$$\mathbf{E}^{scat}=\hat{z}E_z{}^{scat}=\hat{z}\sum_{n=1}^{N}E_{z,n}{}^{scat}(x,y,t) \qquad (2)$$

The electric field components can be computed using the vector potential using the following equation.

$$E_{z,n}{}^{scat}(x,y,t)=-\mu\frac{\partial A_{z,n}{}^{scat}(x,y,t)}{\partial t} \qquad (3)$$

, where

$$A_{z,n}{}^{scat}(x,y,t)=\frac{1}{2\pi}\int_0^{t-\rho_n/c_0}\frac{f_n(\tau)d\tau}{\sqrt{(t-\tau)^2-(\rho_n/c_0)^2}} \qquad (4)$$

Note that the vector potentials are calculated considering the retarded time and $f_n(\tau)$ is the time profile of the *n*th AS, which is the unknown to be solved for.

We choose the integrated form of the governing equation (1), that is,

$$\int_0^t E_{z,n}{}^{inc}(x,y,t)dt-\mu\sum_{n=1}^{N}A_{z,n}{}^{scat}(x,y,t)=0 \qquad (5)$$

and this equation is tested at N observation points.

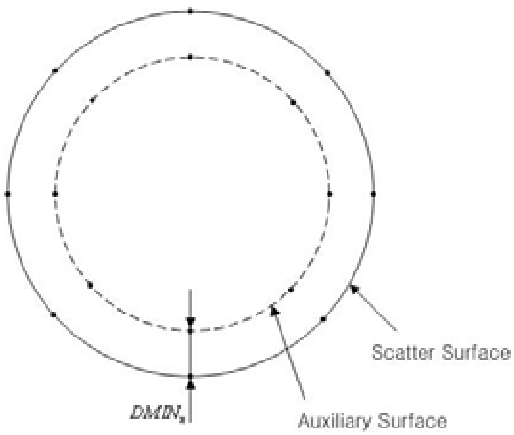

Fig. 2a. AS points and observation points

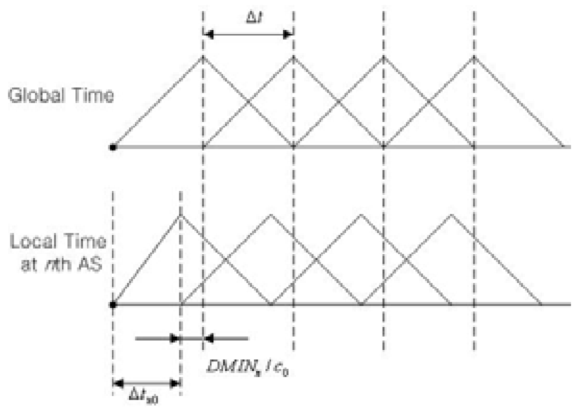

Fig. 2b. Global time and local time at *n*th AS point.

For the purpose of computation, we approximate $f_n(\tau)$ by piecewise linear function, that is, the summation of shifted triangular pulse function as given by

$$f_n(t)=a_{n,1}T_{n0}(t)+\sum_{k=2}^{\infty}a_{n,k}T_k(t-(k-1)\Delta t-\Delta t_{n0}) \qquad (6)$$

, where

$$T_{n0}(t)=\begin{cases} t/\Delta t_{n0} & 0\le t\le \Delta t_{n0} \\ (-t+\Delta t+\Delta t_{n0})/\Delta t & \Delta t_{n0}\le t\le \Delta t_{n0}+\Delta t \end{cases} \qquad (7)$$

and

$$T_k(t)=\begin{cases} 0 & t<-\Delta t \\ t/\Delta t & -\Delta t<t<0 \\ (\Delta t-t)/\Delta t & 0<t<\Delta t \\ 0 & t>\Delta t \end{cases} \qquad (8)$$

In these equation, Δt represents the time step in MOT procedure and Δt_{n0} represents the ascending time of the first pulse function. These treatment is due to the fact that it takes time for the field radiated by the AS to arrive at the testing points and the initial time is set to zero as shown in Fig. 2. In Fig. 2, $DMIN_n$ represents the distance from the nth AS to the nearest test point and $\Delta t_{n0} = \Delta t - DMIN_n / c_0$. From the integral representation of the vector potential, the needed integrations can be performed analytically as follows.

$$\int \frac{d\tau}{\sqrt{(t-\tau)^2 - (\rho_n / c_0)^2}} = -\log\left(t - \tau + \sqrt{(t-\tau)^2 - (\rho_n / c_0)^2} \right) \tag{9}$$

$$\int \frac{\tau d\tau}{\sqrt{(t-\tau)^2 - (\rho_n / c_0)^2}} = \sqrt{(t-\tau)^2 - (\rho_n / c_0)^2} - t\log\left(t - \tau + \sqrt{(t-\tau)^2 - (\rho_n / c_0)^2} \right) \tag{10}$$

To use the implicit scheme for the MOT procedure, we can decompose the vector potential tested at the mth observation point at time $t_k = k\Delta t$ into three components as follows.

$$\begin{aligned} A_z(x_m, y_m, t_k) &= \sum_{\rho_{mn} \le c_0 \Delta t} \frac{1}{2\pi} \int_{t_k - \rho_{mm}/c_0 - \Delta t}^{t_k - \rho_{mn}/c_0} \frac{a_{n,k} T(\tau - (k-1)\Delta t - \Delta t_{n0}) d\tau}{\sqrt{(t-\tau)^2 - (\rho_n / c_0)^2}} \\ &+ \sum_{\rho_{mn} \le c_0 \Delta t} \frac{1}{2\pi} \int_0^{t_k - \rho_{mn}/c_0 - \Delta t} \frac{f_n(\tau) d\tau}{\sqrt{(t-\tau)^2 - (\rho_n / c_0)^2}} \\ &+ \sum_{\rho_{mn} > c_0 \Delta t} \frac{1}{2\pi} \int_0^{t_k - \rho_{mn}/c_0} \frac{f_n(\tau) d\tau}{\sqrt{(t-\tau)^2 - (\rho_n / c_0)^2}} \end{aligned} \tag{11}$$

The first term represents the contribution from the current triangular pulse whose coefficients are yet to be determined and the second term represents the contribution from the retarded pulses within the same spatial range as the first term and the third term represents the contribution from the ASs outside this range, respectively. Then, the governing equation can be cast into matrix equation as

$$[\alpha][a_{n,k}] = [F(t_k)] + [\beta] \tag{12}$$

, where $[\alpha]$ is constructed from the first terms of (11) and $[\beta]$ is constructed from the second and third terms of (11) and $[F(t_k)]$ is constructed from the incident field. Since the matrix $[\alpha]$ is computed only once, the computational burden is not so high. Above process is conceptually right for all k but strictly right for $k \ge 2$ since the first pulses have different representation as shown in (6).

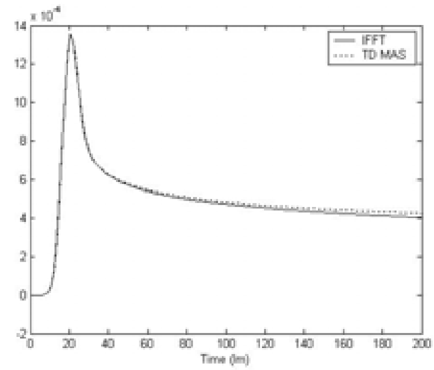

Fig. 3a. Surface currents from time domain MAS

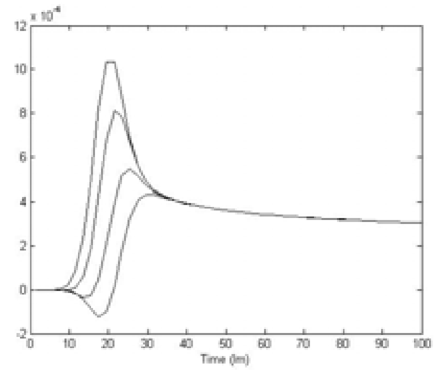

Fig. 3b. Behavior of the ASs

This constitutes the implicit MOT procedure. As is widely known, the time step in the explicit scheme is restricted by the distance of the nearest source points. In contrast to this, the time step in the implicit scheme is solely based on the high-frequency contents in the incident pulse, hence one needs a small number of time steps to obtain a specific duration of time domain results.

4. NUMERICAL RESULTS

To show the validity of the proposed scheme, a simple numerical example is presented. The problem under consideration is scattering by a PEC cylinder of radius 1m illuminated by an incident plane wave going in x direction. Incident plane wave has Gaussian transient profile given by

$$E_z^{inc}(x, y, t) = \frac{4}{\sqrt{\pi}T} e^{-\gamma^2} \tag{13}$$

, where

$$\gamma = \frac{4}{T}\left(t - t_0 - x / c_0\right) \tag{14}$$

and the parameters are $T = 20lm$ and $t_0 = 20lm$. Light meter (lm) is the time needed for the light to travel $1m$. Discretization density is 5 points per wavelength for maximum frequency under consideration. Fig. 3a shows the surface current from the proposed scheme along with that from the inverse Fourier transformed result. For the inverse Fourier transformed result, 1024 frequency samples between 0 and 102.4 MHz were taken and the IFFT (inverse fast Fourier transform) was performed. The two results agree well and the time domain result remains stable. The result from the explicit MOT scheme suffers from the instability and is not shown here. Fig. 3b shows the behavior of some ASs. In time domain MAS, coefficients of ASs are obtained first and the surface currents are calculated as post processing as mentioned before.

5. CONCLUSIONS

In this paper, we presented the implicit MOT procedure for the time domain MAS. Numerical results show that the proposed scheme provides stable results. And due to the large time step, computational cost is much reduced also.

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