Q Evaluation of Small Insulated Antennas in a Lossy Medium and Practical Radiation Efficiency Estimation

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***Abstract*— Quality factors (*Q*) of small insulated antennas in a lossy dielectric medium are evaluated using Chu's equivalent circuits of TM_{01} and TE_{01} spherical waves. Using the evaluated *Q*, the upper limit of radiation efficiency for the required bandwidth is calculated for the link loss estimation. The result shows that a magnetic type antenna has much higher *Q* and the advantage in its radiation efficiency is sacrificed at low frequency for the required bandwidth.**

***Index Terms*— Lossy medium, *Q*, radiation efficiency, small antenna.**

I. Introduction

Recently, with the demand of a wireless system in the medical area, RF transmitters are implanted or swallowed into the human body which is considered as a lossy medium. For such a RF transmitter in a lossy medium, powered by battery, the link loss should be investigated thoroughly to optimize its output power, and the radiation efficiency of the antenna becomes an important factor of the link loss estimation. Moreover, in case of the capsule endoscopy that is swallowed and transmits the images of the intestines [1], the required bandwidth of the communication system should be broad enough to transmit high data rate images, so that the Q of the antenna should be also examined.

Especially, in this letter, an antenna for the capsule endoscopy is focused on. An antenna of the small capsule endoscopy in the human body can be approximately modeled as an omni-directional antenna insulated by a lossless sphere in a lossy medium as depicted in Fig. 1. For such an antenna modeled in Fig. 1, there is an applicable study on its fundamental limitation of the radiation efficiency [2]. On the Q of an antenna in a lossy medium, there is a general study for the evaluation of Q from the field energy stored in the medium [6], in which Q of a bare dipole antenna in a lossy medium was evaluated and verified by the value obtained from its impedance as an example. The objective of this letter is the evaluation of the minimum Q, under the maximum radiation efficiency condition, of an unspecified antenna insulated by a lossless sphere in a lossy medium as modeled in Fig. 1. The evaluated Q is needed for the calculation of the actual radiation efficiency of an antenna adjusted to satisfy the required bandwidth.

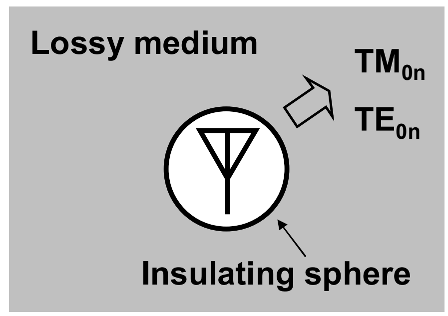


Fig. 1. Model of an omni-directional antenna of the capsule endoscopy in the human body

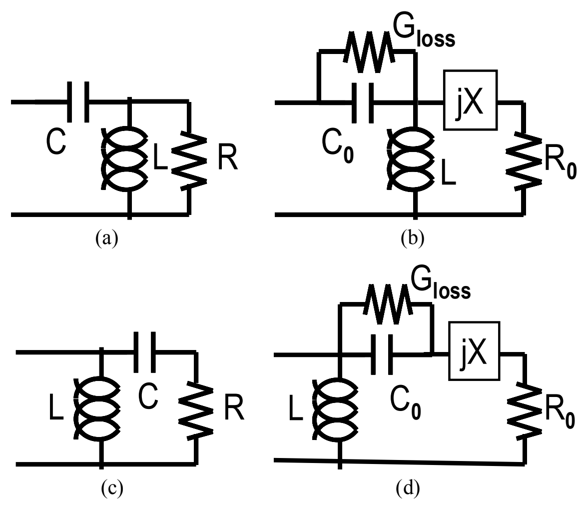


Fig. 2. Equivalent circuits of (a), (b) TM_{01} and (c), (d) TE_{01} spherical waves in a lossy medium

On the minimum Q of an unspecified antenna in free space, Chu presented equivalent circuits of the spherical wave functions [3] and Collin derived the analytic solutions of their Q directly from the stored energy in the field over the whole space [4]. However, for an antenna in a lossy medium, Collin's solutions cannot be applied directly. In this letter, Chu's equivalent circuit of spherical wave is used to evaluate the Q of the antenna in a lossy medium. Only TM_{01} or TE_{01} wave is considered for the evaluation of the minimum Q under the maximum radiation efficiency condition [2], [3]. From the result of the Q, the actual radiation efficiency is calculated taking into account the additional loss necessary to satisfy the required bandwidth.

II. Definition of Q and Its Derivation

According to [3], for an unspecified antenna, the minimum Q is achieved when the excited wave is only TM_{01} or TE_{01}. For an antenna in a lossy medium, [2] shows that the maximum radiation efficiency is achieved when the excited wave is only TM_{01} or TE_{01} for an electric or magnetic type antenna, respectively. Thus, we will consider the Q of TM_{01} or TE_{01} wave to evaluate the minimum Q of an electric or magnetic type antenna in a lossy medium under the maximum radiation efficiency condition.

The equivalent circuits are shown in Fig. 2 (a), (c) outside a sphere for TM_{01} and TE_{01} wave, in which the circuit elements have the values of $C=\varepsilon a$, $L=\mu_0 a$, and $R=\eta$, with a being the radius of the sphere [5]. When these circuits are applied to a lossy medium, the capacitor is equivalent to the parallel combination of a lossless capacitor, C_0, and a conductance, G_{loss}. And the radiation resistance, R, is equivalent to the series connection of a reactance, jX, and a resistance, R_0, as shown in Fig. 2. (b), (d) by (1a) and (1b).

$$j\omega C = j\omega\varepsilon a = j\omega\left(\varepsilon_0\varepsilon_r - j\frac{\sigma}{\omega}\right)a = \sigma a + j\omega\varepsilon_0\varepsilon_r a = G_{loss} + j\omega C_0 \tag{1a}$$

$$R = \eta = \mathrm{Re}[\eta] + j\,\mathrm{Im}[\eta] = R_0 + jX \tag{1b}$$

, where $G_{loss}=\sigma a$, $C_0=\varepsilon_d\varepsilon_0 a$, $R_0=\mathrm{Re}[\eta]$, and $X=\mathrm{Im}[\eta]$.

Q's of these circuits are given by the conventional definition [4]

$$Q_W(\omega) = \begin{cases} \dfrac{2\omega W_e}{P} &, W_e > W_m \\ \dfrac{2\omega W_m}{P} &, W_e < W_m \end{cases} \tag{2}$$

,where W_m and W_e are the stored magnetic and electric energy, and P is the dissipated power. For the equivalent circuits of Fig. 2, (2) is given as

$$Q_W(\omega) = \begin{cases} \dfrac{-1}{\mathrm{Re}[Z(\omega)]}\cdot\mathrm{Im}\left[\dfrac{1}{j\omega C}\right] &, TM_{01}\text{ wave} \\ \dfrac{-1}{\mathrm{Re}[Y(\omega)]}\cdot\mathrm{Im}\left[\dfrac{1}{j\omega L}\right] &, TE_{01}\text{ wave} \end{cases} \tag{3}$$

, where $Z(\omega)$ and $Y(\omega)$ are the impedance and admittance of the TM_{01} and TE_{01} equivalent circuits. In (3), the stored energy in the reactance, jX, is neglected assuming Q is high and the stored energy in L and C is much higher than that.

According to [3], Q of (2) is equal to the reciprocal of the fractional bandwidth of the equivalent circuit when its Q is high. Therefore, for the verification of the Q of (3), Q defined from the half-power bandwidth (HPBW) of the matched reflection coefficient is calculated and compared [6].

$$Q_\Gamma(\omega_0) = \frac{2\omega_0}{\text{HPBW of }\Gamma(\omega)} \tag{4}$$

, where $\Gamma(\omega)$ is the reflection coefficient with matching at ω_0.

III. Practical Radiation Efficiency

The radiation efficiency of spherical waves in a lossy medium was derived analytically in [2]. We can also obtain the exactly same radiation efficiencies of TM_{01} and TE_{01} waves using the equivalent circuits of Fig. 2 as

$$\eta_{eff} = \begin{cases} \dfrac{\mathrm{Re}\left[\left(\dfrac{1}{R}+\dfrac{1}{j\omega L}\right)^{-1}\right]}{\mathrm{Re}[Z(\omega)]}e^{2|\mathrm{Im}[k]|a} &, TM_{01}\text{ wave} \\ \dfrac{\mathrm{Re}[R]}{\mathrm{Re}\left[R+\dfrac{1}{j\omega C}\right]}e^{2|\mathrm{Im}[k]|a} &, TE_{01}\text{ wave} \end{cases} \tag{5}$$

, where $k = \omega\sqrt{\varepsilon\mu}$.

The exponential term in (5) is included for the evaluation of the gain as the product of (5) and the directivity, in accordance with [2]. The conventional field calculation using the gain assumes that the field starts from the origin point, neglecting antenna size. So the attenuation loss which does not actually exist inside the lossless sphere should be compensated with this exponential term.

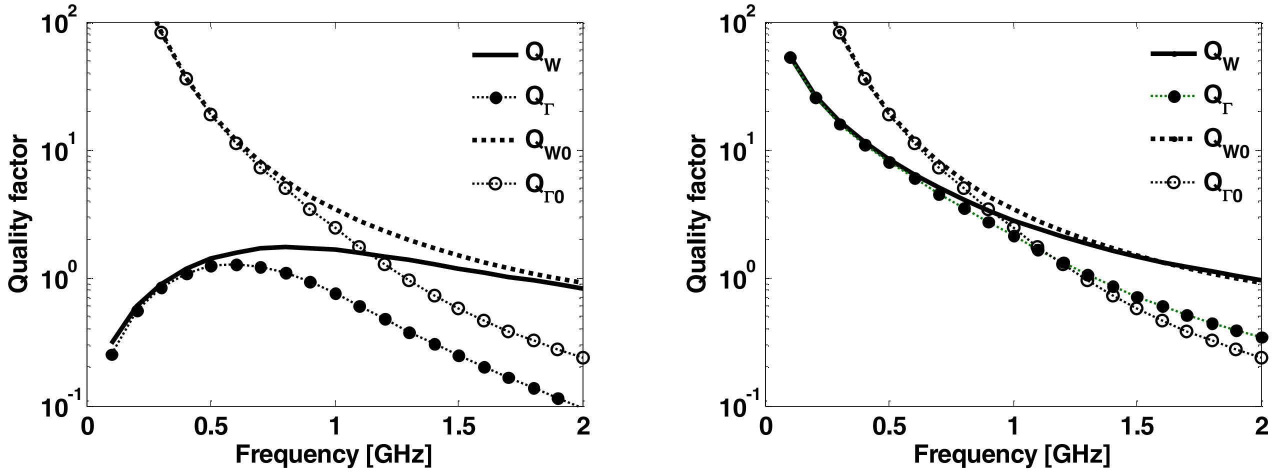
(5) is given by considering only outside the sphere on the assumption that there is no loss inside the sphere. But if the Q of an antenna is higher than the required Q, we should insert some loss inside the sphere to decrease it. The total radiation efficiency considering this additional loss can be expressed as following.

The original Q and radiation efficiency of the problem is given as

$$Q_{\text{intrinsic}} = \frac{2W}{P_{rad}+P_{loss.out}} \quad , \quad \eta_{eff.\text{intrinsic}} = \frac{P_{rad}}{P_{rad}+P_{loss.out}} \tag{6}$$

, where W is the stored magnetic or electric energy, P_{rad} is the radiating power, and $P_{loss.out}$ is the dissipated power outside the sphere in a lossy medium.

After inserting the additional loss, $P_{loss.in}$, inside the sphere to obtain the required Q, the resulting radiation efficiency becomes



(a) (b)

Fig. 3. Q's of (a) TM_{01} and (b) TE_{01} waves outside the 5 mm-radius sphere in a lossy (σ=1) and lossless (σ=0) medium of ε_r=55. Q_W and Q_{W0} are by the conventional definition of (3). Q_Γ and $Q_{\Gamma 0}$ are by (5) using the HPBW of matched reflection coefficient. The zero in subscript letter means lossless case.

$$\eta_{eff.total} = \frac{P_{rad}}{P_{rad} + P_{loss.out} + P_{loss.in}} = \eta_{eff.\mathrm{intrinsic}} \cdot \frac{Q_{\mathrm{intrinsic}}}{Q_{required}} \quad (7)$$

, where $Q_{required} = \dfrac{2W}{P_{rad} + P_{loss.out} + P_{loss.in}}$.

Therefore, the radiation efficiency considering the additional loss to satisfy the required Q is

$$\eta_{eff.total} = \begin{cases} \eta_{eff.\mathrm{intrinsic}} & , \quad Q_{\mathrm{intrinsic}} \le Q_{required} \\ \eta_{eff.\mathrm{intrinsic}} \cdot \dfrac{Q_{\mathrm{intrinsic}}}{Q_{required}} & , \quad Q_{\mathrm{intrinsic}} > Q_{required} \end{cases} . \quad (8)$$

IV. Application Example

We evaluated the minimum Q's under the maximum radiation efficiencies for the case of unspecified antennas which is inside the 5 mm-radius sphere, being insulated from the outside lossy medium of ε_r=55 and σ=1. The permittivity and conductivity values are approximately determined from the average dielectric properties of the human body tissue [7].

For the evaluation of (4), a series inductor to the input of the circuit of Fig. 1 (b) and a parallel capacitor to that of Fig. 1 (d) were added respectively as the matching element. And, for reference, the Q's of the same antennas in a lossless medium are also calculated, which are Q_{W0} and $Q_{\Gamma 0}$ using (2) and (4). Q_{W0} becomes equal to as in [8]

$$Q_{W0} = \frac{1}{k_0 a} + \frac{1}{(k_0 a)^3} \quad (9)$$

, where k_0 is the wave number of a lossless medium.

Fig. 3. (a) and (b) are the comparing results of (3), (4), and (9) for the electric and magnetic type antenna in the frequency of 0.1~2 GHz. It shows that the result of (3), Q_W, approaches the reciprocal of the HPBW when an antenna is small, although its value is small for the TM_{01} case. As expected, the Q of the magnetic type antenna is higher than that of the electric type antenna due to the higher radiation efficiency as shown in [2], and diverges to infinity as the frequency goes to zero. This high Q at low frequency is inappropriate when the required Q is lower than that. E.g., Fig. 4 shows the case. The required Q in Fig. 4 is given from the required matched bandwidth of 100MHz by (4). In this case, at the frequencies where the evaluated Q is higher than the required value, by adding a parallel resistance, G_a, to the input of the circuit, the resulting Q is made equal to the required value. The value of the resistance, G_a, is given by

$$G_a = \mathrm{Re}[Y(\omega)] \cdot \left(\frac{Q_{\mathrm{intrinsic}}}{Q_{required}} - 1 \right) \quad [\mathrm{S \cdot m}] \cdot \quad (10)$$

And the resulting radiation efficiency is obtained by (8). With this resistance, G_a, $Q_{\Gamma,G\ added}$ by (4) is depicted in Fig. 4, showing it satisfying the required value.

Fig. 5 illustrates the original radiation efficiencies by (5) and the practical radiation efficiency to satisfy the required Q by (8) using the result of the intrinsic Q. It shows the advantage of a magnetic type antenna to an electric type antenna in their radiation efficiency decreases at low frequency due to its higher Q, when considering the required bandwidth.

V. Conclusion

The Q's of the small insulated antennas in a lossy medium was presented and compared with the reciprocal of the matched HPBW. The evaluated Q's represent the minimum values under the maximum radiation efficiencies. From the Q-value, the practical radiation efficiency for the required bandwidth was

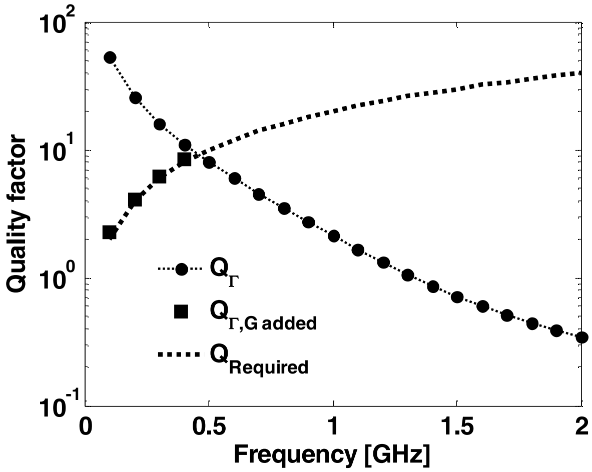


Fig. 4. Comparison of original, reduced, and required Q of TE_{01} wave. Required Q is given with the matched HPBW=100MHz. The radius of lossless sphere = 5 mm, and the medium properties of ε_r=55, and σ=1.

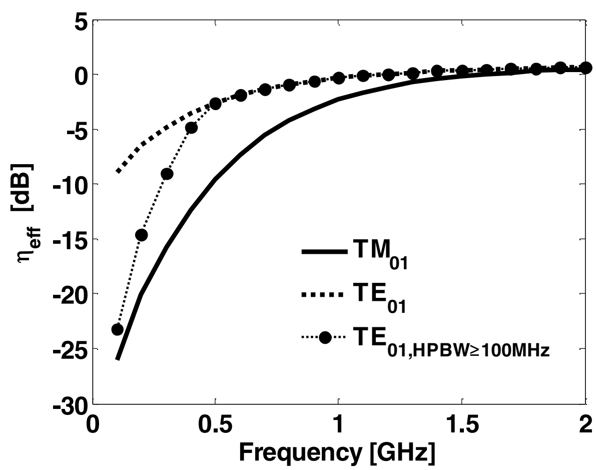


Fig. 5. Radiation efficiencies of TM_{01} and TE_{01} waves considering the required matched HPBW=100MHz. The radius of lossless sphere = 5 mm, and the lossy medium properties are ε_r=55, and σ=1.

calculated, which is a critical factor of the link loss in the use of a small magnetic type antenna in a lossy medium.

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References

[1] G. Iddan, G. Meron, A. Glukhovsky, and P. Swain, "Wireless capsule endoscopy," *Nature*, vol. 405, no. 6785, pp. 417, May 2000

[2] A. Karlsson, "Physical limitations of antennas in a lossy medium," *IEEE Trans. Antennas Propagat.*, vol. 52, pp. 2027-2033, Aug. 2004.

[3] L. J. Chu, "Physical limitations of omni-directional antenna," *J. Appl. Phys.*, vol. 19, pp. 1163–1175, Dec. 1948.

[4] R. E. Collin and S. Rothschild, "Evaluation of antenna Q," *IEEE Trans. Antennas Propagat.*, vol. AP-12, pp. 23–27, 1964.

[5] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961, p. 279.

[6] A. D. Yaghjian, "Improved formulas for the Q of antennas with highly lossy dispersive materials," *IEEE Antennas Wireless Propag. Lett.*, vol. 5, pp. 365–369, 2006.

[7] http://www.fcc.gov/fcc-bin/dielec.sh

[8] J. McLean, "A reexamination of the fundamental limits on the radiation Q of electrically small antennas," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 672–676, 1996.