Method of Auxiliary Sources for the Analysis of Insulated Wire Antennas

Yonghoon Kim*(1), Kyung Heon Koo(2), and Sangwook Nam(3)
(1)Samsung Electronics Co., Ltd., Korea
(2)Department of Electronics Engineering, University of Incheon, Korea
(3)School of Electrical Engineering and INMC, Seoul National University, Korea
E-mail: yhkim@aelsn.ac.kr

Introduction

A possibility of applying the method of auxiliary sources (MAS) in analyzing the characteristics of insulated wire antennas in layered media is investigated. As seen in the previous works, the MAS can be successfully applied to the analysis of wire antennas of bare type in (concentrically) layered media [1]. This is because the modeling of those configurations is mathematically easy with a help of well-established dyadic Green’s functions (DGF) for layered media. But there has not been much effort to apply analytical method for the analysis of insulated antennas in layered media even though these problems are often met in practical applications such as implanted antennas or the transmitter antenna of wireless capsule endoscopy [2]. In this paper, using the standard MAS formulation for scattering analysis, the existing DGF-MAS technique is extended to analyze the wire antenna with a spherical insulation in a layered medium semi-analytically.

Mathematical Formulation

To describe the extended version of the original DGF-MAS technique, the geometry of the problem, an insulated dipole antenna located in a layered medium, is shown in Fig. 1a. There is an insulator, with electric permittivity \( \varepsilon \) and magnetic permeability \( \mu \), in the innermost region of a spherically layered medium. According to the original DGF-MAS, the dipole antenna is modeled considering a set of auxiliary sources (i.e. elementary dipoles) distributed along its axis since the diameter of the antenna is usually substantially less than the radiation wavelength [1]:

\[
J_a (r' = r_q^a) = \sum_{q=1}^{N_a} \delta_q^a I_q^a (r_q^a) \delta (r' - r_q^a)
\]

where \( N_a \) is the number of the auxiliary source points and \( I_q^a (r_q^a) \) is the unknown coefficient of the \( \delta_q^a \)-oriented \( q \)th auxiliary source at \( r' = r_q^a \). Now, two sets of auxiliary sources (i.e. pairs of elementary dipoles) are additionally required for the solution of this problem using DGF-MAS technique, which is same as the standard MAS for the scattering problems [3]. One is a set of auxiliary sources for the region \( C \) (inside the insulation) and they are radiating in the homogeneous medium filled by the material of the insulation in the absence of the layered medium, and located along the antenna axis as well as on the auxiliary surface as...
shown in Fig. 1b. The second is a set of auxiliary sources for the region 1 (the innermost region of the layered medium) radiating in the absence of the insulation, but the insulation region is filled by the material of the region 1, and located on the auxiliary surface as shown in Fig. 1c. To complete the equivalence, the two auxiliary sources are situated at one auxiliary source point on the auxiliary surface such that they are tangent to the auxiliary surface and are orthogonal with each other:

\[
\mathbf{J}_i(\mathbf{r}'=\mathbf{r}'_q) = \sum_{q=1}^{N_i} (\hat{\mathbf{r}}_{q1}^i + \hat{\mathbf{r}}_{q2}^i) \delta(\mathbf{r}'-\mathbf{r}'_q), \quad i = c \text{ or } l
\]

, where \(N_i\) is the number of the auxiliary source points and \(\hat{\mathbf{r}}_{q1}^i, \hat{\mathbf{r}}_{q2}^i\) are the first and second unit tangent vectors at the \(q\)th auxiliary source point for the region \(i\). \(I_{q1}^i, I_{q2}^i\) are the unknown currents flowing in \(\hat{\mathbf{r}}_{q1}^i\) and \(\hat{\mathbf{r}}_{q2}^i\) directions, respectively. Then, the total electromagnetic field for the region \(C\) can be computed from

\[
\mathbf{E}_e(\mathbf{r}) = i\omega \mu_i \sum_{q=1}^{N_i} [I_{q1}^e(\mathbf{r}_q^e) \mathbf{G}_{eo}(\mathbf{r}, \mathbf{r}_q^e) \cdot \hat{\mathbf{z}}_q^e + \sum_{q=1}^{N_i} [I_{q1}^e(\mathbf{r}_q^e) \mathbf{G}_{eo}(\mathbf{r}, \mathbf{r}_q^e) \cdot \hat{\mathbf{r}}_{q1}^i + I_{q2}^e(\mathbf{r}_q^e) \mathbf{G}_{mo}(\mathbf{r}, \mathbf{r}_q^e) \cdot \hat{\mathbf{r}}_{q2}^i]
\]

, here \(\mathbf{G}_{eo} (\mathbf{G}_{mo})\) is the free-space (dyadic Green’s function for the electric (magnetic) field). Similarly, from the new set of auxiliary sources, the electromagnetic field for the region 1 is expressed as

\[
\mathbf{E}_e(\mathbf{r}) = i\omega \mu_i \sum_{q=1}^{N_i} [I_{q1}^e(\mathbf{r}_q^e) \mathbf{G}_{eo}(\mathbf{r}, \mathbf{r}_q^e) \cdot \hat{\mathbf{r}}_{q1}^i + I_{q2}^e(\mathbf{r}_q^e) \mathbf{G}_{mo}(\mathbf{r}, \mathbf{r}_q^e) \cdot \hat{\mathbf{r}}_{q2}^i]
\]

, where \(\mathbf{G}_{eo}^{(1)} (\mathbf{G}_{mo}^{(1)})\) is the dyadic Green’s function for the spherically layered medium with both the source and field point in region 1 [4]. The total magnetic field for the region 1 also can be obtained by the same process. To calculate the unknown coefficients of auxiliary sources in (3)-(5), boundary conditions at the antenna conducting surface (including the feeding gap) and the insulation surface are imposed:

\[
\hat{\mathbf{r}}_p^a \cdot \mathbf{E}_e(\mathbf{r}_p^a) = \begin{cases} 0, & \text{if } \mathbf{r}_p^a \text{ is on the conducting surface} \\ -\frac{V_g}{d}, & \text{if } \mathbf{r}_p^a \text{ is in the feeding gap} \end{cases}, \quad p = 1,2,\ldots,N_a
\]

\[
\hat{\mathbf{r}}_{p1} \cdot \mathbf{E}_e(\mathbf{r}_{p1}^a) = \hat{\mathbf{r}}_{p2} \cdot \mathbf{E}_e(\mathbf{r}_{p2}^a) = \hat{\mathbf{r}}_{p3} \cdot \mathbf{E}_e(\mathbf{r}_{p3}^a) = \hat{\mathbf{r}}_{p4} \cdot \mathbf{E}_e(\mathbf{r}_{p4}^a), \quad p = 1,2,\ldots,N_f = \frac{N_e + N_l}{2}
\]

, where \(\hat{\mathbf{r}}_{p1}^a\) is the unit tangent vector at the \(p\)th testing point at the antenna surface and \(\hat{\mathbf{r}}_{p1}, \hat{\mathbf{r}}_{p2}\) are the first and second unit tangent vectors at the \(p\)th testing point. The testing points are selected just the same manner except that they are located at the physical insulation surface. Then the resultant matrix equation is given by

\[
[Z][I] = [V]
\]

, where
Now the matrix equation in (8) can be solved for the unknown coefficients and the field values and the quantity of interests such as the surface current density and the input impedance of antenna can be computed from these values.

**Numerical Results**

To check the validity of the proposed extended DGF-MAS technique, a dipole antenna with a spherical insulation \( a \) which is concentric to a spherically layered medium as shown in Fig.1a is analyzed. The dipole antenna with \( L=1 \text{cm} \) and \( d=2w=0.2 \text{mm} \) is simulated at 300 MHz. The calculated impedances for various medium parameters are presented in Table I along with those from the original DGF-MAS technique. The two results agree well and this proves the accuracy of the proposed extended DGF-MAS approach. In simulations, \( N_a=257 \), \( N_c=N_t=222 \) (10 grids), \( \Delta \text{a} = 0.3a \), and \( \Delta \text{c} = 0.5a \). (\( \Delta \text{a} \) and \( \Delta \text{c} \) represent the distance between the insulation surface and the auxiliary surface for the region 1 and \( C \), respectively.)

**Conclusions**

In this paper, a possibility of applying the method of auxiliary sources (MAS) in analyzing the characteristics of insulated wire antennas in layered media has been explored. The existing DGF-MAS technique for the analysis of bare antenna in layered media has been revisited and extended to analyze insulated antennas, which has been overlooked by other researchers. Numerical examples have been presented for several interaction problems to demonstrate validity, accuracy, and efficiency of the proposed technique. The proposed method can also be applied to the analysis of antennas immersed in arbitrary shaped insulation layer. Furthermore, analysis of the wire antenna with a spherically layered insulator which is eccentric to a spherically layered medium and more practical case of insulated antenna problem such as the wire antenna with a spherically layered insulator in a cylindrically layered medium, which models the wireless capsule endoscopy application roughly, is under investigation.

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References:


Fig. 1. An insulated antenna with a insulation in spherical $N$-layer medium. ($N=3$)
(a) Original model. (b) Simulated equivalence for region $C$. (c) Simulated equivalence for region 1.

TABLE I. Calculated input impedance of dipole antenna with a spherical insulation in spherically layered medium

<table>
<thead>
<tr>
<th>Medium parameters</th>
<th>Antenna input impedance ($Z_{in}=R_{in}-iX_{in}$)</th>
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<tbody>
<tr>
<td></td>
<td>Extended DGF-MAS (PROPOSED)</td>
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<tr>
<td>Insulation in free space</td>
<td>$a_1=10$ mm, $\varepsilon_1=(57.6+i33.15)\varepsilon_0$, $\varepsilon_1=\varepsilon_0$.</td>
</tr>
<tr>
<td>Insulation in 2-layer medium</td>
<td>$a_1=20$ mm, $\varepsilon_1=5+i$, $\varepsilon_2=2$.</td>
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