Mode-Based Analysis of Resonant Characteristics for Near-Field Coupled Small Antennas

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***Abstract*—This letter proposes a new approach for estimating resonant characteristics of near-field coupled small antennas. It is based on the equivalent circuit representation of the interaction of two antennas using the addition theorem of spherical modes. Using the proposed method, the splitting of the resonant frequency and the resonant impedance of the near-field coupled small antennas can be obtained by the impedance characteristics of the isolated antenna. The results are shown to be in good agreement with the full electromagnetic (EM) simulation.**

***Index Terms*—Addition theorem, coupled mode theory (CMT), electrically small antenna (ESA), near-field coupling, wireless power transfer.**

I. Introduction

WIRELESS power transfer is the basic principle of radio frequency identification (RFID) and near-field communication (NFC) systems [1]. When two or more antennas are placed closely, they are coupled to each other, and the energy of each antenna can then be transferred. It has been recently demonstrated that power can be transferred with high efficiency between two strongly coupled self-resonant coils [2].

The space outside of an antenna can be divided into the near-field region and the far-field region. The near-field region is further divided into the reactive-field region and the radiating-field region [3]. In the reactive near-field region, the energy is stored and oscillated in space but is not radiated. When the resonant evanescent fields of two antennas overlap, they are strongly coupled to each other, and this coupling can be modeled by using the coupled mode theory (CMT) [4].

According to the CMT, the split resonant frequencies are given as

$$\omega = \frac{\omega_1 + \omega_2}{2} \pm \sqrt{\left(\frac{\omega_1 - \omega_2}{2}\right)^2 + |k|^2} \tag{1}$$

where ω_1 and ω_2 are the resonant frequencies of each stand-alone resonator and k is the coupling coefficient [5]. If two resonators are identical, two resonant frequencies are shifted above and below the original resonant frequency by k and classified as the odd mode and the even mode resonant frequencies according to the direction of resonant currents. When the antennas are strongly coupled, the input impedance of the transmitting antenna at the fixed frequency also varies depending on the operating conditions, such as the distance between and the relative orientation of the coupled antennas.

Recently, the frequency splitting and the input impedance for coupled electrically small antennas (ESAs) in parallel have been investigated based on the results of full electromagnetic (EM) simulations and measurements [6]. According to the study, the input impedance of the source antenna at the odd-mode resonant frequency is given as

$$R_{\mathrm{in,odd}} \approx R_{\mathrm{load}} + 2R_{\mathrm{loss}} \tag{2}$$

where the load impedance and the ohmic resistance are denoted as R_{load} and R_{loss}. Therefore, if two antennas are operated at odd-mode resonant frequency by using frequency tracking, the input impedance is almost same to the fixed load impedance, and the coupled antennas can be easily matched to each other irrespective of the operating conditions without the difficulty of having to change the matching impedance.

However, the odd-mode resonant frequency varies according to the distance and the orientation of the antennas. Therefore, it is necessary to find an effective way to characterize the coupled ESAs for the power transfer system in the odd-mode resonant condition.

II. Coupling Analysis Using the Addition Theorem

When an antenna is placed in free space, the field can be represented by spherical waves [7]. Hence, the coupling of the antenna can be considered as interactions of spherical modes. This interaction can be analyzed using the addition theorem. By using the addition theorem, the wave functions in one coordinate system are linearly translated in terms of the other coordinate system [8]. When the antenna is electrically small, the antenna generates predominantly the TE_{10} or TM_{10} mode, depending on the antenna configuration [9]. Hence, the coupling between two ESAs can be analyzed by the interaction of the TE_{10} or TM_{10} mode of each antenna.

If two electric dipoles exist in space (one is a z-directed dipole at its origin, and the other is a dipole with an arbitrary orientation $\mathbf{U}_0$ at P, as shown in Fig. 1), the normalized mutual coupling impedance induced by the interaction of the TM_{10} modes is represented as (3), shown at the bottom of the next page, where η is free-space admittance and $\mathbf{U}_0 = \mathbf{x}_0 \mathrm{u}_1 + \mathbf{y}_0 \mathrm{u}_2 + \mathbf{z}_0 \mathrm{u}_3$ is a unit vector representing the orientation of the second antenna [10]. Additionally, if the magnetic dipoles exist in the same manner,

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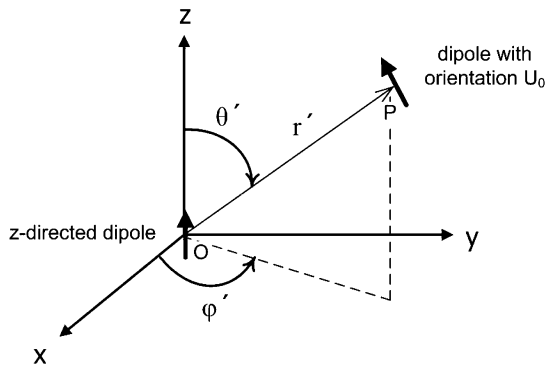

Fig. 1. Two coupled dipoles with arbitrary orientation [10].

the normalized mutual impedance induced by the interaction of TE_{10} modes is the same as in (3) due to the duality of the addition theorem [8, p. 595]. When the TM_{10} and TE_{10} modes interact with each other, the normalized mutual impedance can be given as [10]

$$\bar{\mathrm{Z}}_{12(\mathrm{TM10-TE10})} = \frac{3\hat{\mathrm{H}}_1^{(2)}(kr')\sin\theta'\{\sin\varphi'\mathrm{u}_1 - \cos\varphi'\mathrm{u}_2\}}{2jkr'}. \tag{4}$$

In general, the calculation of the mutual coupling impedance using the addition theorem is valid only under the specific condition that the minimum hypothetical sphere that can include the entire structure of the single antenna does not touch the other sphere. In the case of the ESAs, however, the minimum required distance between the spheres is electrically quite small. Hence, the antennas are strongly coupled before touching the spheres. Therefore, although the use of the addition theorem imposes a limitation on the physical size of the antenna, it is still a useful method to analyze the coupling between ESAs of wireless power transfer systems.

When two antennas are coupled, the input impedance of the source antenna with load impedance Z_{load} at the load antenna is given as

$$Z_{\mathrm{in}} = Z_a - \frac{Z_{12}^2}{Z_a + Z_{\mathrm{load}}} = Z_a - \frac{\bar{Z}_{12}^2 R_r^2}{Z_a + Z_{\mathrm{load}}} \tag{5}$$

where the impedance and the radiation resistance of the standalone antenna are denoted as Z_a and R_r.

Once the input real and imaginary impedance of the standalone antenna are obtained using the EM simulation, the input impedance of the source antenna can be evaluated using (5) for any location or orientation of the load antenna. At this time, the radiation resistance is given as being the input resistance of the isolated antenna by the EM simulation with perfect electric conductor (PEC) [6]. The resonant frequencies can be found using (5) because the reactance of the input impedance crosses the zero-axis at the resonant frequency. Accordingly, the resonant frequencies for the various distances and orientations can be evaluated without any additional EM simulation.

III. Example

As an example, the proposed method is applied to the center-fed 15-turn small helix antennas, which are made of copper wire. The dimensions of the antenna are height of 0.071 m and radius of 0.08 m, as shown in Fig. 2. The self-resonant frequency of the antenna is approximately 13.56 MHz.

For the helix antenna, the magnitude of the current distribution can be assumed to be a cosine function along the wire, as in [2]. When the current distribution is given, the normalized power flowing away from the antenna can be calculated as presented in [11]. The normalized magnitude of each spherical mode can be found using

$$\begin{bmatrix} \boldsymbol{b}'_\beta \\ \cdots \\ \boldsymbol{b}''_\beta \end{bmatrix} = \mathrm{Y}_0^{-1/2} \int \begin{bmatrix} \vec{\boldsymbol{M}}^{(1)}_{e(o)mn} \\ \cdots \\ \vec{\boldsymbol{N}}^{(1)}_{e(o)mn} \end{bmatrix} \cdot J dv \tag{6}$$

where $\boldsymbol{J}$ is the current density, Y_0 is the admittance of free space, and $\vec{\boldsymbol{M}}^{(1)}_{e(o)mn}$ and $\vec{\boldsymbol{N}}^{(1)}_{e(o)mn}$ are the spherical vector wave function with the spherical Bessel function of the first kind. The normalized magnitude and flowing power for each spherical mode of the antenna are presented in Table I. From the table, it can be shown that most power radiates in the TE_{10} and TM_{10} modes. Therefore, the total radiation resistance of the antenna can be approximated as the sum of the radiation resistances of the dominant TE_{10} and TM_{10} modes, and the coupling of the small helix antennas can be modeled by the interaction of the TE_{10} and TM_{10} modes generated from each antenna, as previously described. Therefore, the mutual impedance of the antennas can be given as

$$\begin{aligned} Z_{12} &= \left(R_{r(\mathrm{TE10})} + R_{r(\mathrm{TM10})}\right) \cdot \bar{\mathrm{Z}}_{12(\mathrm{TM10})} \\ &\quad + \left(2\sqrt{R_{r(\mathrm{TE10})} R_{r(\mathrm{TM10})}}\right) \cdot \bar{\mathrm{Z}}_{12(\mathrm{TM10-TE10})} \\ &\approx R_{r(\mathrm{tot})} \cdot \bar{\mathrm{Z}}_{12(\mathrm{TM10})} \end{aligned} \tag{7}$$

if the coupling by the interaction between the different modes is small enough to be ignored.

The split resonant frequencies of the coupled small helix antennas for different configurations are compared in Fig. 3. One is the result from the full EM simulation, which includes the entire coupling structure of two antennas, and the other is calculated by

$$\bar{\mathrm{Z}}_{12(\mathrm{TM10})} = \frac{3}{2}\left[\frac{2\eta\hat{\mathrm{H}}_1^{(2)}(kr')\cos\theta'\{\cos\varphi'\sin\theta'\mathrm{u}_1 + \sin\varphi'\sin\theta'\mathrm{u}_2 + \cos\theta'\mathrm{u}_3\}}{kr'^2\omega\varepsilon} - \frac{\dot{\hat{\mathrm{H}}}_1^{(2)}(kr')\sin\theta'\{\cos\varphi'\cos\theta'\mathrm{u}_1 + \sin\varphi'\cos\theta'\mathrm{u}_2 - \sin\theta'\mathrm{u}_3\}}{kr'}\right] \tag{3}$$

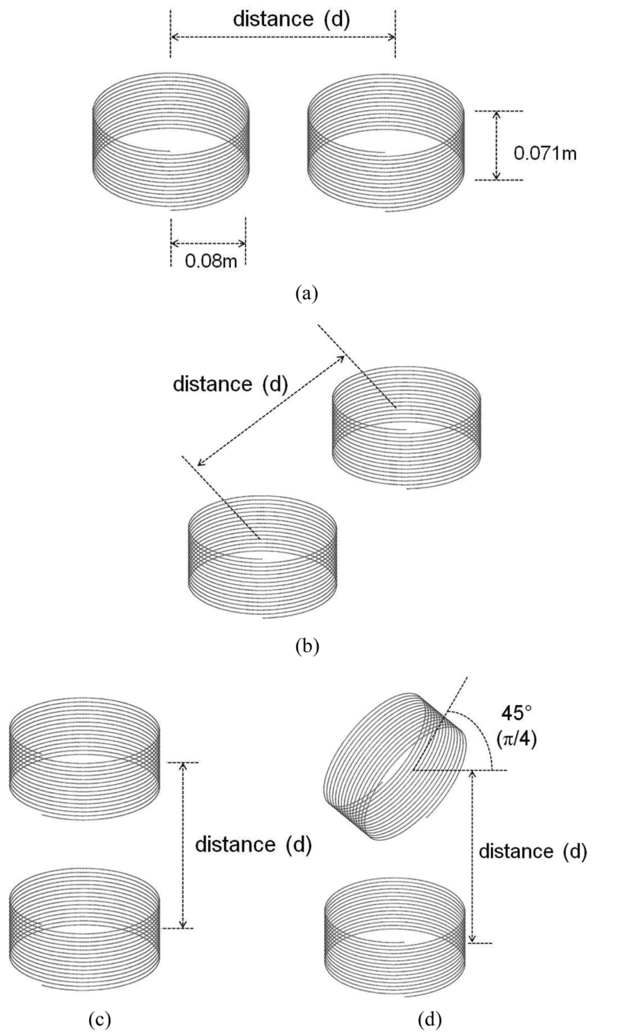

Fig. 2. Two coupled small helix antennas: (a) parallel configuration, (b) diagonal configuration, (c) collinear configuration, and (d) $\pi/4$-tilted collinear configuration.

TABLE I
CHARACTERISTICS OF SPHERICAL MODES FOR THE SMALL HELIX ANTENNA

Mode	Normalized Magnitude $\lvert b_\beta \rvert$	Normalized Flowing Power $\lvert b_\beta \rvert^2/2$	Mode	Normalized Magnitude $\lvert b_\beta \rvert$	Normalized Flowing Power $\lvert b_\beta \rvert^2/2$
TE_{10}	9.80×10^{-2}	4.80×10^{-3}	TM_{10}	8.10×10^{-2}	3.30×10^{-3}
TE_{11}	1.44×10^{-6}	1.03×10^{-12}	TM_{11}	4.80×10^{-3}	1.15×10^{-5}
TE_{21}	3.65×10^{-7}	6.66×10^{-14}	TM_{21}	4.45×10^{-7}	9.88×10^{-14}
TE_{22}	3.21×10^{-6}	5.17×10^{-12}	TM_{22}	1.06×10^{-5}	5.60×10^{-11}

our proposed method. The proposed method uses the impedance characteristics, which are obtained from the EM simulation of the standalone single antenna, and the mutual impedance is calculated using the proposed method. The relative error for estimating the split resonant frequencies is less than 2% for most of cases, and the maximum error is 4.5% for the closest parallel case. Therefore, the proposed method can be used for estimating the operating frequency of the wireless power transfer by using frequency tracking.

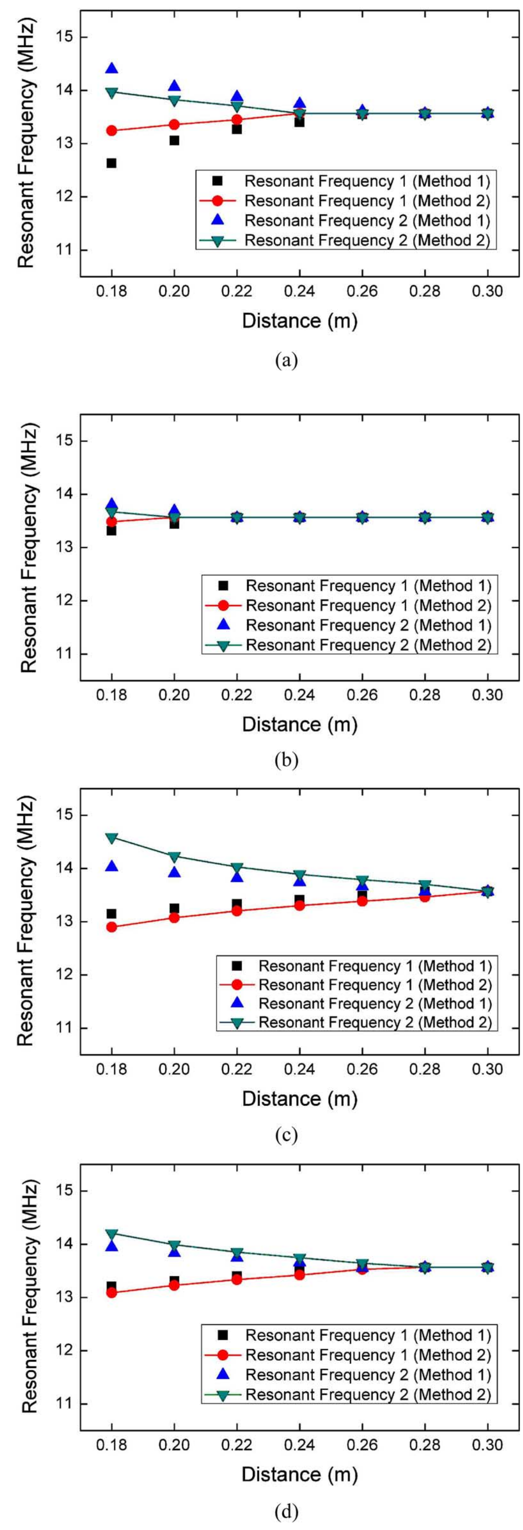

Fig. 3. Comparison of the resonant frequencies for two coupled small helix antennas: (a) parallel configuration, (b) diagonal configuration, (c) collinear configuration, and (d) $\pi/4$-tilted collinear configuration. (Method 1: the full EM simulation; Method 2: the proposed method based on the addition theorem).

Fig. 4 shows the comparison of the input resistance of two coupled small helix antennas at the resonance frequencies. The results are evaluated by using the full EM simulation and the

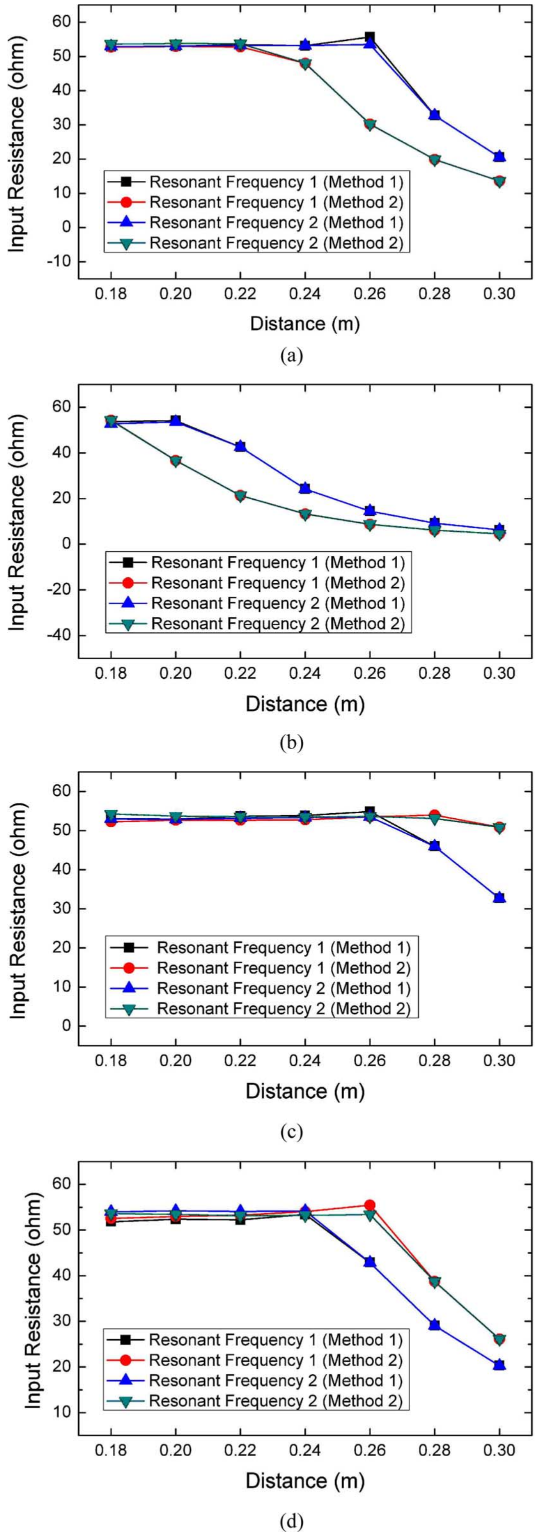

Fig. 4. Comparison of the input resistance at the resonant frequencies for two coupled small helix antennas: (a) parallel configuration, (b) diagonal configuration, (c) collinear configuration, and (d) $\pi/4$-tilted collinear configuration. (Method 1: the full EM simulation; Method 2: the proposed method based on the addition theorem).

proposed method. When the antennas are strongly coupled and the resonant frequencies are split, the input resistance at the resonant frequency is approximately equal to the load impedance, as in [6].

The differences between the results using full EM simulation and the proposed method are thought to be due to the neglect of the interaction between the TE_{10} and TM_{10} mode and coupling of other higher order modes.

IV. Conclusion

In this letter, a mode-based analysis method for the coupling of ESAs is proposed. The coupling of antennas can be modeled as the interaction of fields generated from each antenna, and it is calculated by using the addition theorem. In the case of an ESA, the radiated field is dominantly composed of the TE_{10} and TM_{10} modes. Hence, the coupling of small antennas can be calculated based on the interaction of the TE_{10} or TM_{10} modes using the addition theorem. By using the proposed method, the resonant frequency and resonant impedance characteristics between two ESAs in any position and direction can be obtained by using the simulation of the isolated single antenna only.

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