

New Design Formulas for Impedance-Transforming 3-dB Marchand Baluns

Hee-Ran Ahn, *Senior Member, IEEE*, and Sangwook Nam, *Senior Member, IEEE*

Abstract—New design formulas for impedance-transforming 3-dB Marchand baluns are proposed. They are about the even- and odd-mode impedances of the coupled transmission-line sections of the Marchand baluns and determined by coupling coefficients together with termination impedances. The particular property proposed in this paper is to choose the coupling coefficient arbitrarily, resulting in infinite sets of design formulas available. This is quite different from the conventional design approach in which only one coupling coefficient is possible. For the perfect isolation of the Marchand balun, an isolation circuit (IC) is needed, being composed of two 90° transmission-line sections and resistance(s). Sufficient area to build such a long IC is, however, inherently not available. For this, ways to reduce the IC size are also suggested. To validate them, a microstrip Marchand balun terminated in 130 and $70\ \Omega$ is designed at a design center frequency of 1.5 GHz and tested. The measured results are in good agreement with prediction, showing that power divisions are 3.57 and 3.262 dB, return losses at all ports are better than 21 dB, and the isolation is better than 20 dB around the design center frequency. The measured phase difference between two balanced signals is $180^\circ \pm 2^\circ$ in about 50% bandwidth.

Index Terms—Compact isolation circuits (ICs) of Marchand baluns, impedance-transforming 3-dB Marchand baluns, modified Π - and T -types of equivalent circuits of transmission-line sections, new design formulas for Marchand baluns.

I. INTRODUCTION

THE MARCHAND balun introduced in 1944 [1] has been realized in various transmission-line structures and utilized for various applications such as balanced mixers, balanced amplifiers, and frequency multipliers [2]–[23]. The Marchand balun has two sets of coupled transmission-line sections, and if the even- and odd-mode impedances of the two sets of transmission-line sections are identical, 180° phase difference between two balanced signals can be achieved theoretically in whole frequencies (except even multiples of the design center frequency). Due to such attractive performance, the Marchand balun has received substantial attention from microwave circuit designers. In the beginning of 2000, design equations were developed for impedance-transforming 3-dB Marchand baluns and have been used for more than one decade [2], [4], [6], [7],

[9], [11], [15]–[18], [20]–[23]. Since the coupling coefficient of the coupled transmission-line sections suggested by the conventional design approach is, however, determined by the termination impedances, only one value is possible and it can be 0 or -3 dB. It is desirable that the coupling coefficients be determined by the coupling structures [24]–[28] and independent of the termination impedances of the Marchand baluns.

In this paper, new design formulas of the even- and odd-mode impedances are derived for the impedance-transforming 3-dB Marchand baluns. Since the coupling coefficient of the coupling transmission-line sections can be selected arbitrarily, infinite sets of design formulas of the even- and odd-mode impedances are available. It will be shown later that the conventional design equations [2], [4], [6], [7], [9], [11], [15]–[18], [20]–[23] are only one among the infinite sets of even- and odd-mode impedances proposed in this paper. Any impedance-transforming 3-dB balun may be equivalent to a circuit consisting of two impedance transformers and an isolation circuit (IC), which may be determined by the phase delay of the impedance transformers [24]. In the case of the Marchand balun, since the phase delay of the impedance transformers are 90° or -90° , the IC should be composed of two 90° transmission-line sections and resistance(s) [24]. Since the two output ports (balanced ports) of the Marchand baluns are, however, located close to each other, no sufficient area to construct such a long IC is available. The 90° transmission-line sections might be shortened by using the conventional Π - [26], [29] or T -type of equivalent circuit, but the resulting bandwidth may be reduced. To avoid the bandwidth reduction, a transmission-line section is divided by the number of N and the corresponding design equations of Π - or T -type of equivalent circuit are derived. In the case of infinite number of N , the T -type equivalent circuit with N is very similar to the artificial transmission-line approach [30], but different fundamentally with some number of N . To validate the design equations of the Marchand baluns and the modified equivalent circuits of a transmission-line section, a microstrip Marchand balun terminated in 130 and $70\ \Omega$ is fabricated and measured.

II. IMPEDANCE-TRANSFORMING MARCHAND BALUNS

A. Description and Conventional Design Formulas

A Marchand balun is described in Fig. 1(a) where it consists of two sets of coupled transmission-line sections. It is terminated in R_r at port ① and R_L at ports ② and ③ for the impedance transforming, and an internal port ④ is located between two sets of coupled transmission-line sections. For the perfect matching at ports ② and ③ and the perfect isolation between the two ports, an IC is needed to be connected. For the analyses, a set of coupled transmission-line sections terminated

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The authors are with the School of Electrical Engineering and Computer Science, Seoul National University, Seoul 151-742, Korea (e-mail: hranahn@gmail.com; snam@snu.ac.kr).

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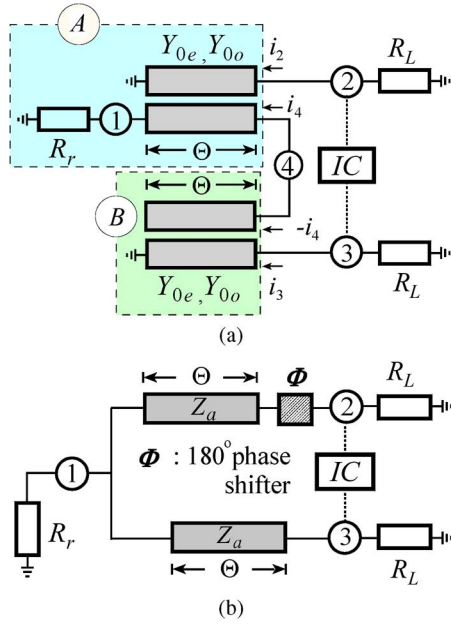


Fig. 1. Impedance-transforming baluns. (a) Marchand balun. (b) Equivalent circuit of the Marchand balun only at a design center frequency.

in R_r at port ① is designated as A and another as B , and an equivalent circuit being valid only at a design center frequency is introduced in Fig. 1(b).

The conventional design formulas of the even- and odd-mode impedances Z_{0ec} , Z_{0oc} and its coupling coefficient C_c [2], [4], [6], [7], [9], [11], [15]–[18], [20]–[23] are

$$Z_{0ec} = R_r \sqrt{\frac{1 + C_c}{1 - C_c}} \quad (1a)$$

$$Z_{0oc} = R_r \sqrt{\frac{1 - C_c}{1 + C_c}} \quad (1b)$$

$$C_c = \sqrt{\frac{R_r}{R_r + 2R_L}} \quad (1c)$$

The conventional coupling coefficient in (1c) is a function of the termination impedances R_r and R_L . It may be -3 dB when $2R_L = R_r$ and 0 dB when $R_r \gg 2R_L$. The coupling coefficient should be determined by the structure of the coupled transmission-line sections itself, not by the termination impedances [24]–[28]. Therefore, the design equations in (1) should be modified so that the coupling coefficients can be independent of the termination impedances. The two sets of coupled transmission-line sections will be synthesized for the design formulas.

B. Design Formulas

If the currents at ports ②, ③, and ④ are denoted as i_2 , i_3 , and i_4 , as shown in Fig. 1(a), and the voltages between ports ② and ④ and ground as v_2 and v_4 , the relation between currents i_2 , i_4 and voltages v_2 , v_4 are

$$\begin{bmatrix} i_2 \\ i_4 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} v_2 \\ v_4 \end{bmatrix} \quad (2)$$

where

$$A_{11} = -j \frac{Y_{0e} + Y_{0o}}{2} \cot \theta + \frac{R_r (Y_{0o} - Y_{0e})^2 \csc^2 \theta}{4D_A} \quad (2a)$$

$$A_{12} = A_{21} = -j \frac{Y_{0o} - Y_{0e}}{2} \cot \theta + \frac{R_r (Y_{0o}^2 - Y_{0e}^2) \csc^2 \theta}{4D_A} \quad (2b)$$

$$A_{22} = -j \frac{Y_{0e} + Y_{0o}}{2} \cot \theta + \frac{R_r (Y_{0o} + Y_{0e})^2 \csc^2 \theta}{4D_A} \quad (2c)$$

with

$$D_A = 1 - jR_r \left(\frac{Y_{0e} + Y_{0o}}{2} \right) \cot \theta$$

where A_{ij} is the admittance parameters contributed by ports ② and ④ of the A circuit in Fig. 1(a).

If v_3 is the voltage between port ③ and ground, the admittance parameters of the coupled transmission-line sections B contributed by ports ④ and ③ are

$$\begin{bmatrix} -i_4 \\ i_3 \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} v_4 \\ v_3 \end{bmatrix} \quad (3)$$

where

$$B_{11} = j \frac{(Y_{0e} + Y_{0o}) \tan \theta}{2} \quad (3a)$$

$$B_{12} = B_{21} = j \frac{(Y_{0o} - Y_{0e}) \tan \theta}{2} \quad (3b)$$

$$B_{22} = -j \frac{Y_{0e} + Y_{0o}}{2} \cot \theta + j \frac{(Y_{0o} - Y_{0e})^2 \csc^2 \theta}{Y_{0e} + Y_{0o}} \quad (3c)$$

where B_{ij} are two-port admittance parameters in terms of ports ④ and ③ of the B circuit in Fig. 1(a).

From (2) and (3), with port ① terminated in R_r , the admittance parameters Y_{ij} contributed by ports ② and ③ without the IC are obtained as

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} \quad (4)$$

where

$$Y_{11} = A_{11} - \frac{A_{21}^2}{B_{11} + A_{22}} \quad (4a)$$

$$Y_{12} = Y_{21} = -\frac{A_{12}B_{12}}{B_{11} + A_{22}} \quad (4b)$$

$$Y_{22} = B_{22} - \frac{B_{12}^2}{B_{11} + A_{22}} \quad (4c)$$

When $\Theta = 90^\circ$, the admittance parameters in (4) are simplified as

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = R_r \left(\frac{Y_{0o} - Y_{0e}}{2} \right)^2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \end{bmatrix} \quad (5)$$

As is well known, if the power is excited at port ① in Fig. 1(a), the power is divided between ports ② and ③ where the two waves are 180° out of phase at a design center frequency. To derive the design formulas of the complicated Marchand baluns easily, an equivalent is needed, as depicted in Fig. 1(b), consisting of two transmission-line sections with the characteristic impedance of Z_a and the electrical length of Θ ,

one 180° phase shifter, and IC. In this case, the electrical length of Θ is 90° at a design center frequency and the characteristic impedance of Z_a is $\sqrt{2R_rR_L}$ [24], [31], [32]. Without the IC, the admittance parameters between ports ② and ③ of the equivalent circuit in Fig. 1(b) are

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \frac{1}{2R_L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \end{bmatrix}. \quad (6)$$

For the two circuits in Fig. 1 to have the same frequency responses at the design center frequency, the admittance parameters in (5) and (6) should be equal to each other, from which the following relation holds:

$$\frac{Y_{0o} - Y_{0e}}{2} = \frac{1}{\sqrt{2R_rR_L}}. \quad (7)$$

The two sets of coupled transmission-line sections should be satisfied with the general definition of the coupling coefficient C such as

$$Y_{0o} = Y_{0e} \frac{1+C}{1-C}. \quad (8)$$

From (7) and (8), the design formulas of the impedance-transforming 3-dB Marchand baluns are obtained as

$$Z_{0e} = \sqrt{2R_rR_L} \frac{C}{1-C} \quad (9a)$$

$$Z_{0o} = \sqrt{2R_rR_L} \frac{C}{1+C}. \quad (9b)$$

In the derived design formulas in (9), infinite sets of the even- and odd-mode impedances may be obtained by varying the coupling coefficient C , and the coupling coefficient is not the function of the termination impedances of R_r and R_L . This is quite different from the conventional one in (1c). The conventional coupling coefficient in (1c) is one of infinite sets in (9) and may be obtained by equating $Z_{0e} = Z_{0ec}$ or $Z_{0o} = Z_{0oc}$ in (1) and (9).

When the termination impedances are $R_r = 50 \Omega$ and $R_L = 100 \Omega$, the conventional coupling coefficient of C_c is only one value of 0.4472 (-6.9897 dB), and the corresponding even- and odd-mode impedances are 80.9 and 30.9 Ω . The same values of the even- and odd-mode impedances are calculated by substituting $C = 0.4472$ into the equations in (9). In addition, there are so many other even- and odd-mode impedances available from the design (9).

C. Frequency Responses of Marchand Baluns

With the termination impedances of $R_r = 50 \Omega$ and $R_L = 100 \Omega$, the even- and odd-mode impedances were calculated by varying the coupling coefficients and are written in Table I. When the coupling coefficient C is -3 dB, the even-mode impedance of Z_{0e} is 242.4 Ω , which is hard to be realized with a planar structure. When C is -5 dB, $Z_{0e} = 128.5 \Omega$ and $Z_{0o} = 36 \Omega$, which may be realizable with microstrip technology. Based on the data in Table I, frequency responses were simulated at a design center frequency of 1 GHz, and the simulation results are plotted in Fig. 2.

Matching at port ①, power divisions of $|S_{21}|$ and $|S_{31}|$ and absolute phase difference between S_{21} and S_{31} are plotted in

TABLE I
EVEN- AND ODD-MODE IMPEDANCES OF MARCHAND BALUNS
WITH $R_r = 50 \Omega$ AND $R_L = 100 \Omega$

C (dB)	-3	-5	-7	-9	-11
$Z_{0e}(\Omega)$	242.4	128.5	80.73	54.99	39.24
$Z_{0o}(\Omega)$	41.45	36	30.88	26.19	21.98

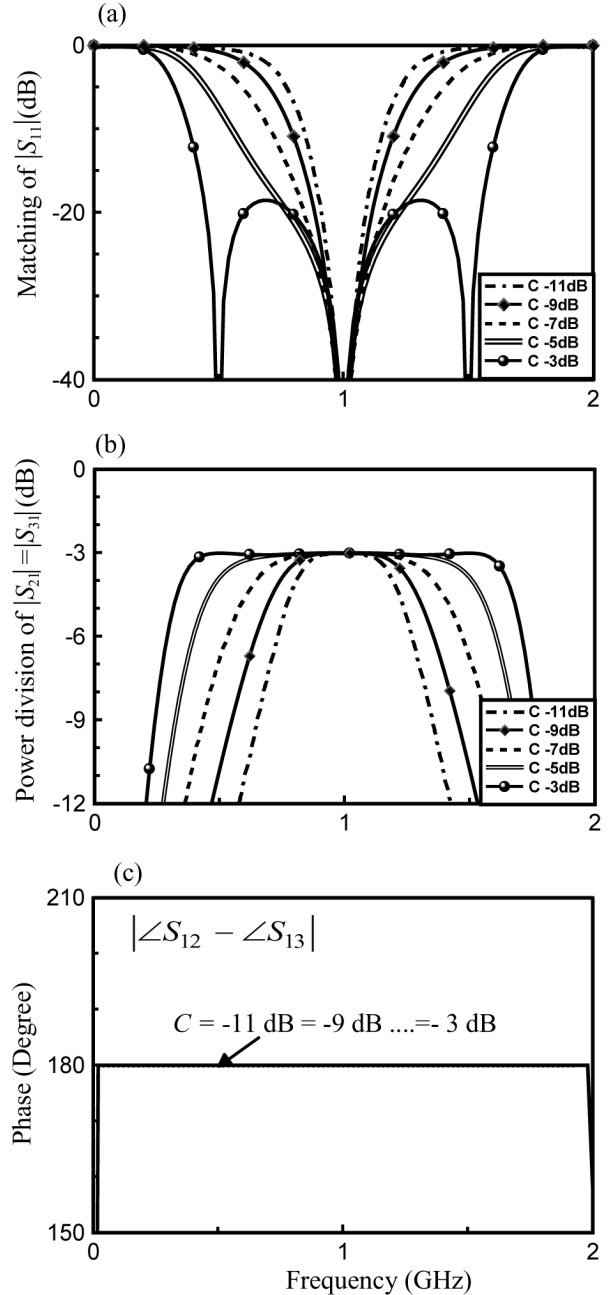


Fig. 2. Simulation results with $R_r = 50 \Omega$, $R_L = 100 \Omega$. (a) $|S_{11}|$. (b) $|S_{21}|$ and $|S_{31}|$. (c) Absolute phase difference between S_{12} and S_{13} .

Fig. 2(a)–(c), respectively. Independently of the coupling coefficients, all are perfectly matched at the design center frequency of 1 GHz [see Fig. 2(a)]. As long as the coupling coefficient of the two coupled transmission-line sections is identical, the frequency responses of S_{21} and S_{31} are identical [see Fig. 2(b)], and the bandwidths increase with the coupling coefficient. The

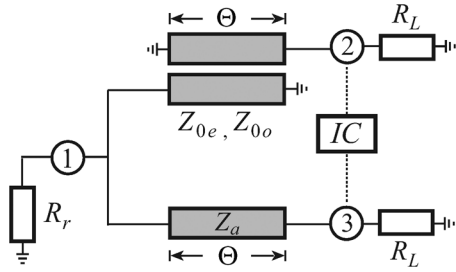


Fig. 3. One circuit being equivalent to that in Fig. 1(b) at a design center frequency.

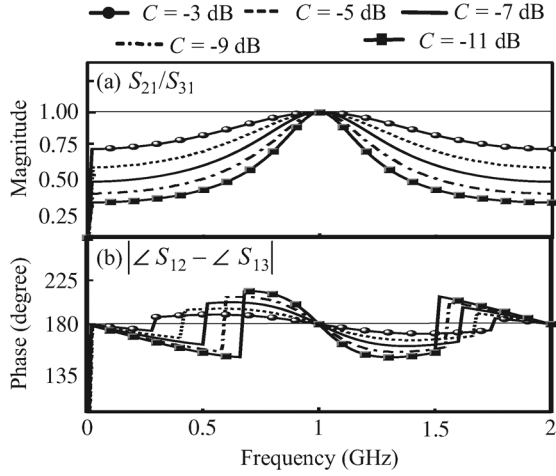


Fig. 4. Frequency responses. (a) Power division ratio of S_{21} to S_{31} . (b) Absolute phase difference between S_{21} and S_{31} .

absolute phase difference between S_{21} and S_{31} [see Fig. 2(c)] is 180° theoretically in whole frequencies, except even multiples of the design center frequency.

D. Properties of Marchand Baluns

There are several ways to realize the equivalent circuit in Fig. 1(b) and one of them is depicted in Fig. 3, where the circuit consists of one set of coupled transmission-line sections with two short circuits in a diagonal direction and one transmission-line section. In this case, the design equations of the coupled transmission-line sections are the same as those in (9).

Under the same condition as the Marchand balun with $R_r = 50 \Omega$ and $R_L = 100 \Omega$ in Fig. 1(a) and Table I, the frequency responses of the equivalent circuit in Fig. 3 were simulated at the design center frequency of 1 GHz, and the simulation results are plotted in Fig. 4 where the magnitude of the ratio of S_{21} to S_{31} is in Fig. 4(a) and absolute phase difference between S_{21} and S_{31} is in Fig. 4(b). The ratio of S_{21} to S_{31} in Fig. 4(a) is unity at the design center frequency for all coupling coefficients, and the difference between S_{21} to S_{31} is gradually bigger with the coupling coefficient smaller and with the operating frequency farther from the center frequency. In contrast to this, the ratio of S_{21} to S_{31} in the Marchand balun is unity independently of the coupling coefficient and the operating frequency [see Fig. 2(b)]. The absolute phase difference [see Fig. 4(b)] between S_{21} and S_{31} of the equivalent circuit in Fig. 3 is 180° only at the design center frequency of 1 GHz for any coupling coefficient, and the deviation from the 180° phase difference is smaller with the

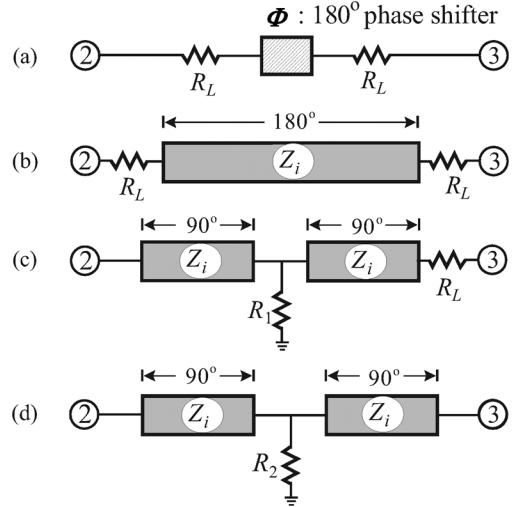


Fig. 5. ICs. (a) One 180° phase shifter and two series isolation resistances. (b) One 180° transmission-line section with two series resistances. (c) Two 90° transmission-line sections with one series and one shunt resistances. (d) Two 90° transmission-line sections with one shunt resistance.

higher coupling coefficient outside of the center frequency. For the Marchand balun in Fig. 2(c), the 180° phase difference between two output signals is achieved independently of the coupling coefficient and the operating frequencies. Due to the attractive properties of the Marchand baluns, they have received substantial attention from circuit designers and have been used for diverse applications for a long time.

III. ICs

A. ICs

Even without the IC (Fig. 1), perfect matching at port ① and perfect power divisions are achieved as shown in Fig. 2, but perfect matching at ports ② and ③ and perfect isolation between ports ② and ③ do not appear. For the perfect balun performance at all ports, the IC is needed. Since the phase delays of S_{21} and S_{31} are 90° and -90° , respectively, the admittance parameters of the IC between ports ② and ③ [see Fig. 1(a)] [24] are

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_{IC} = \frac{1}{2R_L} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}. \quad (10)$$

A circuit with two series resistances of R_L and an 180° phase shifter connected in cascade, as depicted in Fig. 5(a), satisfies the admittance parameters in (10). In addition, the three ICs in Fig. 5 are possible. To realize the 180° phase shifter, one 180° transmission-line section may be employed in Fig. 5(b). Moving one or two resistances of R_L in Fig. 5(b) to the center of the 180° transmission-line section results in the circuit in Fig. 5(c) or (d), respectively. The relation among the characteristic impedance Z_i of the transmission-line section, the output termination impedance of R_L and the resistances of R_1 and R_2 [24] is

$$Z_i^2 = R_1 R_L \quad (11a)$$

$$R_2 = \frac{R_1}{2}. \quad (11b)$$

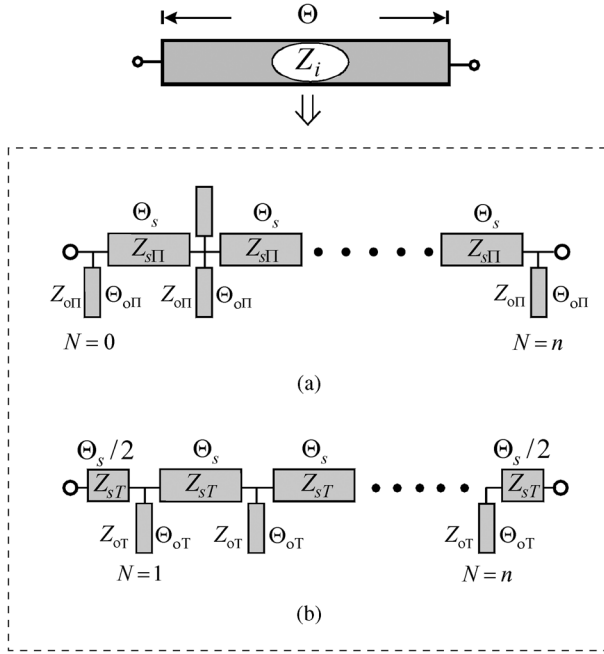


Fig. 6. Modified Π - and T -types of equivalent circuits. (a) Π -type. (b) T -type.

B. Compact ICs

As shown in Fig. 1(a), since two output ports are connected to each other very closely, it is not easy to implement such long ICs in Fig. 5. The transmission-line sections should therefore be reduced for easy fabrication. To shorten the transmission-line sections without changing phase response, modified Π - and T -types of equivalent circuits may be utilized and detailed in Fig. 6 where the characteristic impedance and electrical length of the original transmission section are Z_i and Θ . The modified Π -type equivalent circuit [see Fig. 6(a)] consists of N transmission-line sections with the characteristic impedance of $Z_{s\Pi}$ and $N + 1$ open stubs. The first and last open stubs are the same and the others have two times susceptance of the first or last one. The number of N is therefore equal to the number of the transmission-line sections with $Z_{s\Pi}$. The T -type equivalent circuit [see Fig. 6(b)] is a kind of dual network of the Π -type [see Fig. 6(a)] and composed of N open stubs and $N + 1$ transmission-line sections. The electrical length of the first and last transmission-line sections with the characteristic impedance of Z_{sT} [see Fig. 6(b)] is half of the others and the others are therefore two times of the first or last one. The characteristic impedances of $Z_{o\Pi}$ and Z_{oT} and the lengths of $\Theta_{o\Pi}$ and Θ_{oT} of the open stubs may be chosen arbitrarily as far as their susceptance values of $Y_{o\Pi} \cdot \tan \Theta_{o\Pi}$ and $Y_{oT} \cdot \tan \Theta_{oT}$ are kept the same, where $Y_{o\Pi} = Z_{o\Pi}^{-1}$ and $Y_{oT} = Z_{oT}^{-1}$.

The design equations of the equivalent circuits in Fig. 6 are

$$Z_{s\Pi} = Z_i \frac{\sin \frac{\Theta}{N}}{\sin \Theta_s} \quad (12a)$$

$$Y_{o\Pi} \cdot \tan \Theta_{o\Pi} = \frac{1}{Z_i} \left(\frac{\cos \Theta_s - \cos \frac{\Theta}{N}}{\sin \frac{\Theta}{N}} \right) \quad (12b)$$

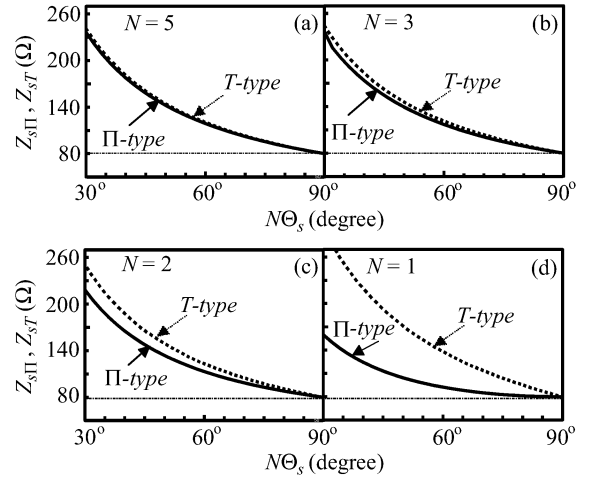


Fig. 7. Characteristic impedance of $Z_{s\Pi}$ and Z_{sT} with $Z_i = 80 \Omega$. (a) $N = 5$. (b) $N = 3$. (c) $N = 2$. (d) $N = 1$.

TABLE II
DESIGN DATA OF Π - AND T -TYPES OF MODIFIED EQUIVALENT CIRCUITS
WITH $Z_i = 80 \Omega$, $\theta = 90^\circ$, AND $N\theta_s = 70^\circ$

N	Π -type		T -type	
	$Z_{s\Pi} (\Omega)$	$S_{u\Pi} (V)$	$Z_{sT} (\Omega)$	$S_{uT} (V)$
$N = 5$	102.187	0.00078	103.195	0.00154
$N = 3$	100.990	0.00130	103.814	0.00254
$N = 2$	98.624	0.00198	105.097	0.00372
$N = 1$	85.134	0.00428	114.252	0.00637

$$Z_{sT} = Z_i \frac{\tan \frac{\Theta}{2N}}{\tan \frac{\Theta_s}{2}} \quad (13a)$$

$$Y_{oT} \cdot \tan \Theta_{oT} = \frac{2}{Z_{sT}} \cdot \frac{Z_{sT} - Z_i \cot \frac{\Theta}{2N} \tan \frac{\Theta_s}{2}}{Z_{sT} \tan \frac{\Theta_s}{2} + Z_i \cot \frac{\Theta}{2N}} \quad (13b)$$

where $\Theta > N\theta_s$, $Y_{o\Pi}$, Y_{oT} are characteristic admittances of open stubs, and Θ_s , $\Theta_{o\Pi}$, Θ_{oT} are electrical lengths of transmission-line sections and open stubs in Fig. 6. The Π -type of the equivalent circuit with $N = 1$ have been used for various applications [24], [26], [29], [33], but that with N greater than 2 has not been discussed yet. For the use of the equivalent circuits, the characteristic impedances of $Z_{s\Pi}$ and Z_{sT} should be realizable. For this, the relation among $Z_{s\Pi}$, Z_{sT} , and N will be studied.

For a transmission-line section with the characteristic impedance of $Z_i = 80 \Omega$ and the electrical length of 90° , the characteristic impedances of $Z_{s\Pi}$ and Z_{sT} were calculated with $N = 5, 3, 2$, and 1, and the calculation results are plotted in Fig. 7 where $N\theta_s$ is the total length of the resulting transmission-line section. When $N = 5$, the characteristic impedances of $Z_{s\Pi}$ and Z_{sT} are about the same with each other [see Fig. 7(a)]. With N smaller, the difference between $Z_{s\Pi}$ and Z_{sT} becomes bigger [see Fig. 7(d)]. In the case of $N\theta_s = 70^\circ$, $Z_{s\Pi}$, $S_{u\Pi}$, Z_{sT} , and S_{uT} are calculated in Table II where $S_{u\Pi} = Y_{o\Pi} \cdot \tan \Theta_{o\Pi}$ and $S_{uT} = Y_{oT} \cdot \tan \Theta_{oT}$ in (12b) and (13b).

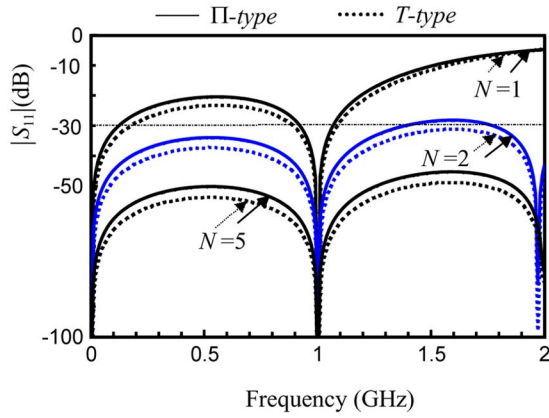

 Fig. 8. Simulation results of $|S_{11}|$.

 TABLE III
 FABRICATION DATA OF A MICROSTRIP MARCHAND BALUN

Coupled transmission-line sections; $C = -10$ dB	
$Z_{0e} = 62.39 \Omega$, $Z_{0o} = 32.41 \Omega$	$w = 1.915$ mm, $s = 0.1655$ mm
Isolation circuit in Fig. 5(d); $R_L = 70 \Omega$ and $R_2 = 51 \Omega$	
$Z_{sT} = 116.5 \Omega$, $\theta_s = 22^\circ$ with $N = 3$	
Impedance transformers	
$R_r = 130 \Omega \rightarrow 50 \Omega$	
CVT _r : 50Ω with 20° long and 97Ω with 44.15° .	
$R_L = 70 \Omega \rightarrow 50 \Omega$	
CVT _L : 50Ω with 30° long and 70.71Ω with 28.56° .	

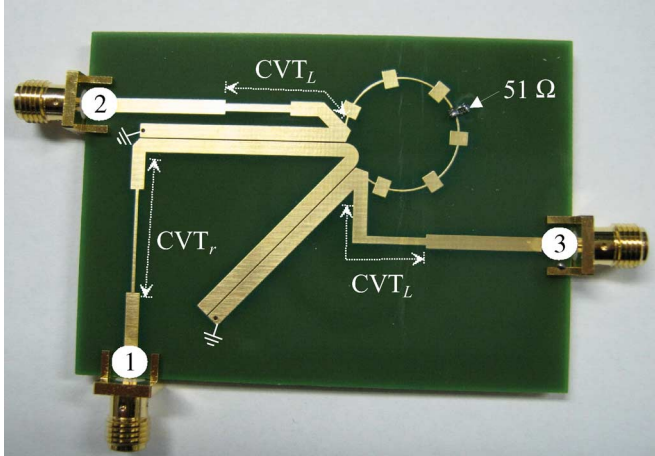
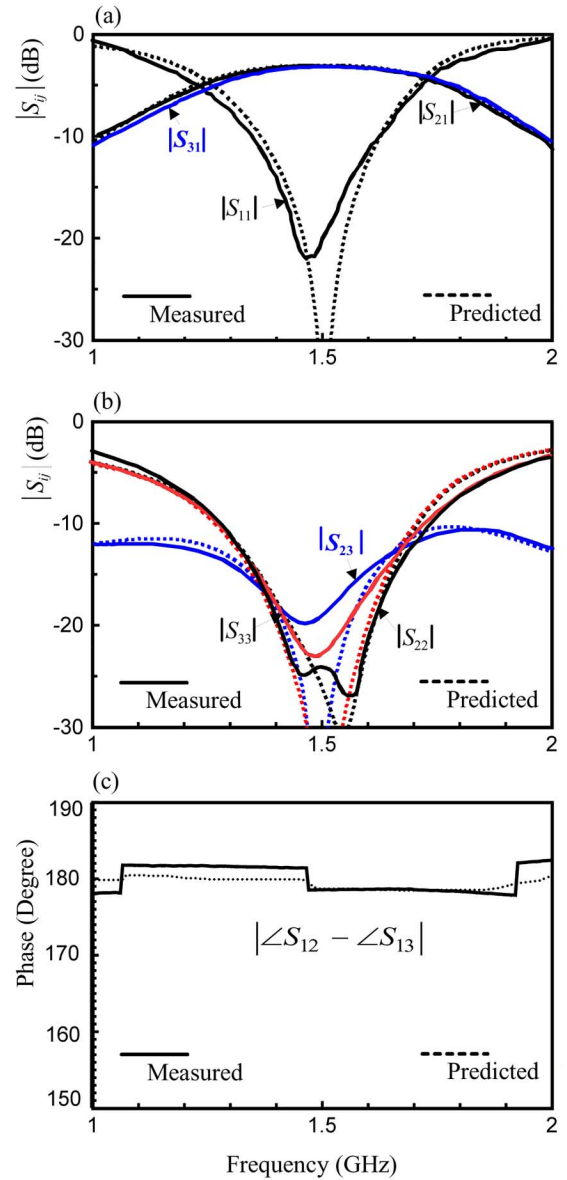


Fig. 9. Fabricated marchand balun.

Based on the data in Table II, the two types of equivalent circuits were simulated at the design center frequency of 1 GHz and ideal capacitances were used for the susceptances produced by the open stubs. The frequency responses of matching performance are plotted in Fig. 8 where the solid and dotted lines are those of the Π - and T -types of the modified equivalent circuits, respectively. When $N = 5$ (Fig. 8), the return loss is more than 50 dB in the entire frequency range of interest. With $N = 2$, the return loss with more than 30 dB is achieved, except several frequencies around 1.6 GHz, and with $N = 1$, poor response is shown in the frequencies higher than 1 GHz. For the use of the 90° transmission-line sections, the number of N should be greater and equal to 2 for good matching performance.


 Fig. 10. Results measured and predicted are compared. (a) Scattering parameters from port ①. (b) Scattering parameters from ports ② and ③. (c) Absolute phase difference between S_{12} and S_{13} .

IV. MEASUREMENTS

To validate design formulas and modified equivalent circuits of a transmission-line section, a microstrip Marchand balun terminated in $R_r = 130 \Omega$ and $R_L = 70 \Omega$ was fabricated on a substrate (FR4, $\epsilon_r = 4.2$, $H = 1$ mm). The dielectric constant of FR4 is found based on a third-order polynomial [34]. $C = -10$ dB and a design center frequency of 1.5 GHz were chosen. The corresponding even- and odd-mode impedances are $Z_{0e} = 62.39 \Omega$ and $Z_{0o} = 32.42 \Omega$, and width w and gap size s of the coupled microstrip transmission-line sections are 1.92 and 0.165 mm, respectively. For the conventional design equations in (1), the coupling coefficient of C_c is -3.174 dB (0.6939) and the even- and odd-mode impedances should be $Z_{0e} = 305.8 \Omega$ and $Z_{0o} = 55.26 \Omega$, which are impossible to be implemented with a microstrip format on any substrate. The IC in Fig. 5(d) was employed and $Z_i = 84.5 \Omega$ in Fig. 5(d) was

calculated for the use of an available chip resistor of $51\ \Omega$. Each 90° transmission-line section was reduced using the modified T -type equivalent circuit to have $N\Theta_s = 66^\circ$ where $N = 3$. The susceptance of S_{uT} produced by one open stub is $0.00280\ \Omega$ and was realized with two open stubs connected in parallel. Each open stub is 4° long and the characteristic impedance is $50\ \Omega$. The two termination impedances are not $50\ \Omega$, and therefore, impedance transformers are needed. To transform $R_r = 130\ \Omega$ into $50\ \Omega$, an asymmetric impedance transformer CVT_r [35], [36] was adopted, and to convert $R_L = 70\ \Omega$ into $50\ \Omega$, CVT_L was employed. The detailed data are summed up in Table III. If the resistance value of R_2 is lowered in Fig. 5(d), then the characteristic impedance of Z_i is lowered from (11), which leads to the size reduction more from the calculation results in Fig. 7.

The fabricated Marchand balun is displayed in Fig. 9, and the results measured and predicted are compared in Fig. 10 where the measured scattering parameters produced from port ① are in Fig. 10(a), those from ports ② and ③ in Fig. 10(b) and absolute phase difference between S_{12} and S_{13} in Fig. 10(c). Measured scattering parameters of the power divisions are -3.57 and -3.62 dB, measured return losses at all ports are better than about 21 dB, and the isolation is better than about 20 dB around the design center frequency of 1.5 GHz. The absolute phase difference between two balanced signals is $180 \pm 2^\circ$ in about 50% bandwidth. The measured results are in good agreement with the prediction, as displayed in Fig. 10.

V. CONCLUSIONS

Conventional design formulas of the even- and odd-mode impedances of the Marchand balun are determined by one coupling coefficient only, being a function of the termination impedances. The coupling coefficient may therefore be 0 or -3 dB, depending on the termination impedances. In general, the coupling coefficient should be determined by the coupling structure itself, not by the termination impedances. To solve the problem, new design formulas are derived by terminating input port with its matched termination impedance. Infinite sets of the even- and odd-mode impedances are possible by varying the coupling coefficient arbitrarily, which allows design flexibility and diverse applications. Since two output ports of the Marchand baluns are placed very closely with each other, no sufficient space for the long IC is available. For this, modified II- and T -types of equivalent circuits are also proposed.

Using the presented design formulas of the Marchand baluns and the modified II- and T -types of equivalent circuits, further applications can be expected.

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Hee-Ran Ahn (S'90–M'95–SM'99) received the B.S., M.S., and Ph.D. degrees in electronic engineering from Sogang University, Seoul, Korea, in 1988, 1990 and 1994, respectively.

Since April 2011, she has been with the School of Electrical Engineering and Computer Science, Seoul National University, Seoul, Korea. From August 2009 to December 2010, she was with the Department of Electrical Engineering, University of California at Los Angeles (UCLA). From July 2005 to August 2009, she was with the Department

of Electronics and Electrical Engineering, Pohang University of Science and Technology (POSTECH), Pohang, Korea. From 1996 to 2002, she was with the Division of Electrical Engineering, Department of Electrical Engineering and Computer Science, Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea. She was also with the Department of Electrical Engineering, Duisburg-Essen University, Duisburg, Germany, where she was involved with the Habilitation dealing with asymmetric passive components in microwave circuits. Her interests include high-frequency and microwave circuit design and biomedical application using microwave theory and techniques. She authored *Asymmetric Passive Component in Microwave Integrated Circuits* (Wiley, 2006).



Sangwook Nam (S'87–M'88–SM'11) received the B.S. degree from Seoul National University, Seoul, Korea, in 1981, the M.S. degree from the Korea Advanced Institute of Science and Technology (KAIST), Seoul, Korea, in 1983, and the Ph.D. degree from The University of Texas at Austin, in 1989, all in electrical engineering.

From 1983 to 1986, he was a Researcher with the Gold Star Central Research Laboratory, Seoul, Korea. Since 1990, he has been a Professor with the School of Electrical Engineering and Computer Science, Seoul National University. His research interests include analysis/design of electromagnetic (EM) structures, antennas, and microwave active/passive circuits.