Efficient Computation of Impedance Matrix in 1-Dimensional Periodic Planar Structures Using Rooftop Functions

Heeduck Chae, Hanseung Lee, Yonghoon Kim, and Sangwook Nam
School of Electrical Engineering and INMC, Seoul National University
E-mail : raptor76@ael.snu.ac.kr

Introduction

There has been much interest in accurate and fast numerical analysis of 1-D periodic planar structures. Recently the efficient method to obtain 1-D periodic Green's functions in planar structure was proposed [1]. Here we propose the efficient method for the evaluation of impedance matrix in applying the method of moment.

Analysis

In typical moment method, the spectral domain expression of impedance matrix for 1-D periodic planar structure with d_x periodicity in x-direction is expressed as [2]

$$Z_{mn} = \frac{1}{2\pi d_x} \sum_{p=-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\mathbf{g}}_m(-k_{xp}, -k_y) \cdot \tilde{\tilde{\mathbf{G}}}(k_{xp}, k_y; z, z') \cdot \tilde{\mathbf{g}}_n(k_{xp}, k_y) dk_y \tag{1}$$

where $k_{xp} (= k_{x0} + 2\pi p / d_x = \beta_{xp} + j\alpha_x)$ is the propagation constant of p-th spatial harmonic and $\tilde{\mathbf{g}}_n(k_{xp}, k_y)$ is the spectral domain basis function of n-th mesh cell. When the rooftop basis function is used, $\tilde{\mathbf{g}}_n(k_{xp}, k_y)$ is written in the form

$$\tilde{g}_{xn}(k_x, k_y) = \Delta_x \Delta_y \,\mathrm{sinc}^2(k_x \tfrac{\Delta_x}{2}) \,\mathrm{sinc}(k_y \tfrac{\Delta_y}{2}) e^{jk_x x_n} e^{jk_y y_n} \tag{2a}$$

$$\tilde{g}_{yn}(k_x, k_y) = \Delta_x \Delta_y \,\mathrm{sinc}(k_x \tfrac{\Delta_x}{2}) \,\mathrm{sinc}^2(k_y \tfrac{\Delta_y}{2}) e^{jk_x x_n} e^{jk_y y_n} \;. \tag{2b}$$

And, in planar structures, the spectral domain Green's function $\tilde{\tilde{\mathbf{G}}}(k_{xp}, k_y; z, z')$ in (1) can be obtained in closed form by the spectral domain immitance approach. Then the Green's function of xx-component is expressed as

$$\tilde{G}_{xx} = -Z_{TE} + k_x{}^2 \frac{(Z_{TE} - Z_{TM})}{k_\rho{}^2} \tag{3}$$

where Z_{TE} and Z_{TM} are the variables used in the immitance approach.

1-4244-0878-4/07/$20.00 ©2007 IEEE

In the numerical point of view, the slow convergence of $\tilde{\tilde{\mathbf{G}}}(k_{xp}, k_y; z, z')$ makes the evaluation of (1) too time-consuming. For the efficient evaluation of (1), the approximated Green's function is extracted from $\tilde{\tilde{\mathbf{G}}}(k_{xp}, k_y; z, z')$. Here the approximated Green's function is obtained using the generalized pencil-of-function (GPOF) method and its *xx*-component expression is given by

$$\tilde{G}_{xx}^{Approx.} = -\sum_{i=1}^{N_A} c_i^A \frac{e^{-s_i^A k_\rho}}{k_\rho} + k_x^{\ 2} \sum_{i=1}^{N_\phi} c_i^\phi \frac{e^{-s_i^\phi k_\rho}}{k_\rho} . \tag{4}$$

Using (4), (1) can be rearranged as

$$Z_{mn} = \frac{1}{2\pi d_x} \sum_{p \in \mathbf{P}_{set}} \int_{-k_{yp,\max}}^{k_{yp,\max}} \tilde{\mathbf{g}}_m(-k_{xp}, -k_y) \cdot \left\{ \tilde{\tilde{\mathbf{G}}} - \tilde{\tilde{\mathbf{G}}}^{Approx.} \right\} \cdot \tilde{\mathbf{g}}_n(k_{xp}, k_y) dk_y + Z_{mn}^{Approx.} \tag{5}$$

where $k_{yp,\max} = \sqrt{k_{\rho,\max}^{\ 2} - \beta_{xp}^{\ 2} + \alpha_x^{\ 2}}$ and $\mathbf{P}_{set} = \{ p \mid 0 \le \sqrt{\beta_{xp}^{\ 2} - \alpha_x^{\ 2}} < k_{\rho,\max} \}$. Since the approximation of (4) is quite accurate in $k_\rho > k_{\rho,\max}$, the required integration region of first term of right side in (5) can be significantly reduced. Though $k_{\rho,\max}$ varies as structure parameters, it is just a several times k_0 .

$Z_{mn}^{Approx.}$ also can be rearranged. The terms related to k_{xp} are maintained in the spectral domain form, while the other terms related to k_y are converted to the spatial domain form using the Parseval's theorem and the table of integration [3]. The final expression of *xx*-component of $Z_{mn}^{Approx.}$ becomes

$$Z_{xx,mn}^{Approx.} = \frac{1}{2\pi d_x} \int_{-\infty}^{\infty} U_{xx}(y - y_n + y_m) V_{xx}(y) dy \tag{6}$$

$$\begin{aligned} & U_{xx}(y - y_n + y_m) \\ & = -2\sum_{i=1}^{N_A} c_i^A \sum_{p=-\infty}^{\infty} \left\{ \Delta_x^{\ 2} \operatorname{sinc}^4(k_{xp} \tfrac{\Delta_x}{2}) e^{jk_{xp}(x_n - x_m)} K_0\left[\sqrt{k_{xp}^{\ 2}} \sqrt{(y - y_n + y_m)^2 + (s_i^A)^2} \right] \right\} \\ & +2\sum_{i=1}^{N_\phi} c_i^\phi \sum_{p=-\infty}^{\infty} \left\{ k_{xp}^{\ 2} \Delta_x^{\ 2} \operatorname{sinc}^4(k_{xp} \tfrac{\Delta_x}{2}) e^{jk_{xp}(x_n - x_m)} K_0\left[\sqrt{k_{xp}^{\ 2}} \sqrt{(y - y_n + y_m)^2 + (s_i^\phi)^2} \right] \right\} \end{aligned} \tag{7}$$

$$V_{xx}(y) = \begin{cases} \Delta_y \left\{ 1 - \left| y / \Delta_y \right| \right\}, & \left| y / \Delta_y \right| \le 1 \\ 0, & otherwise. \end{cases} . \tag{8}$$

In (6), the integration over only finite interval of $2\Delta_y$ is needed due to (8). And also, the summations in (7) are fast convergent series which has the asymptotic form of e^{-ap} / p^k . The other components of $Z_{mn}^{Approx.}$ can be derived similarly and omitted here.

Numerical result

Fig. 1 shows the relative magnitude of integrand (1) and (5) at each spatial harmonic in single microstrip structures with the periodicity of 8 mm and the grayed rectangle indicates the required integration regions that are numerically meaningful. It demonstrates that (1) needs numerical integration beyond $250k_0$ with many harmonics, while the proposed method needs integration over only small interval with a single harmonic to obtain converged solution. Fig. 2 shows the convergence of the summation (7) and p=20 is sufficient for convergence. Using the proposed method, propagation constants were obtained for the structure in Fig. 3 by solving eigenvalue problem with EFIE and the result is shown in Fig. 4. The result using Bloch wave analysis of unit-cell is also displayed, which shows good agreement.

Conclusion

An efficient computation method of impedance matrix in 1-D periodic planar structure was proposed and numerically verified. The extraction of approximated Green's function by GPOF method reduces the required integration region dramatically and the extracted parts are converted to finite integral with fast convergent summation, which enables the fast analysis of 1-D periodic planar structure with sufficient accuracy.

References

[1] P. Baccarelli, C. D. Nallo, S. Paulotto, and D. R. Jackson, "A full-wave numerical approach for modal analysis of 1-D periodic microstrip structures," *IEEE Trans. Microw. Theory Tech*., vol. 54, no. 4, pp. 1350–1362, 2006.

[2] P. K. Potharazu and D. R. Jackson, "Analysis and design of a leaky-wave EMC dipole array," *IEEE Trans. Antennas Propag.,* vol. 40, no. 8, pp. 950-958, 1992.

[3] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, Series, and Products*, Academic Press, San Diego, California, 1996.

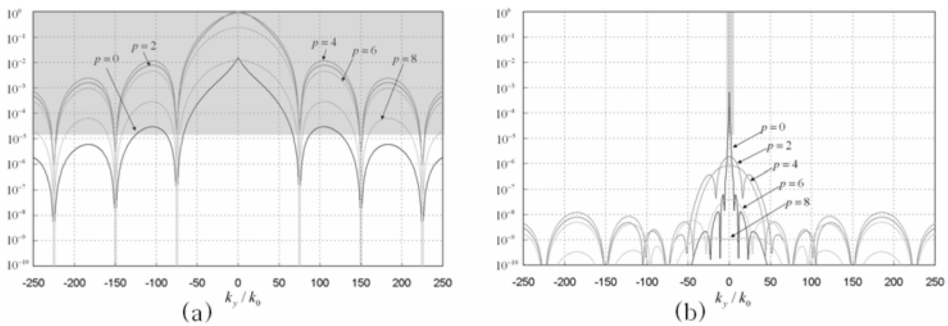

Fig. 1. Comparison of relative magnitude of (a) integrand in (1) and (b) integrand in (5). (Used parameters : $\Delta_x=\Delta_y=0.5mm$, $(x_m-x_n, y_m-y_n)=(2\Delta_x,\Delta_y)$, $d_x=8mm$, $f=8GHz$, $\beta_{x0}/k_0=1.12$, $\alpha/k_0=0.2$, $\varepsilon_r=2.52$, $h=0.504mm$)

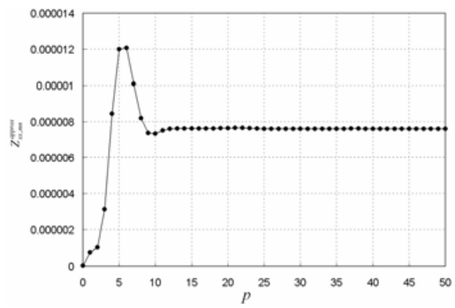

Fig. 2. Convergence of summation (7). (Used parameters are same as in Fig. 1.)

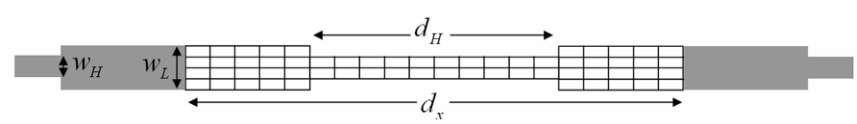

Fig. 3. Geometry of the unit-cell in 1-D periodic structure
($d_H=8mm$, $d_x=16mm$, $w_L=1.4mm$, $w_H=0.7mm$, $\varepsilon_r=2.52$, $h=0.504mm$)

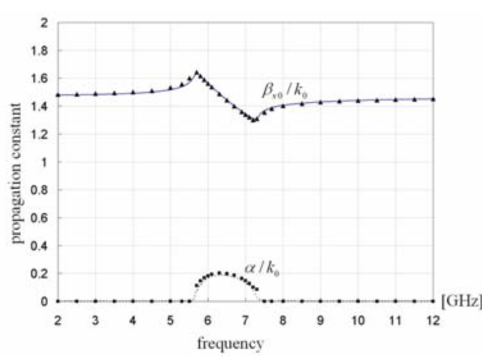

Fig. 4. Propagation constants for the structure in Fig. 3. (Triangle and circle-proposed method, solid and dashed line-result of Bloch wave analysis)