# Analysis of Periodically Loaded Structure and Application to Leaky Wave Antennas

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## 1. Introduction

A periodically L-C or C loaded transmission line has repetitive passband and stopband. The fast wave, that has the phase constant $\beta$ which is smaller than the free-space wave number $k_0$, exists in the passband nearby the stopband. If the fast wave propagates along the transmission line, it simultaneously generates the leaky wave to the free space with a particular beam direction related to the phase constant like the Cherenkov radiation. Thus, by changing the phase constant $\beta$ according to frequency or loaded reactance, the beam direction of leaky wave can be controllable [1]. A full wave simulation to find the leaky regions of sufficiently large array, that can be assumed like periodic, is heavily time consuming. Alternative easy and simple method is Bloch wave analysis [2]. Using Bloch wave analysis, the propagation constant can be easily and quickly calculated using Floquet mode theorem and ABCD parameters of the unit cell [3], [4].

In here, we describe the Bloch wave analysis of the periodically loaded transmission line and design two leaky wave antennas. The one uses defected ground structure (DGS) as array elements of leaky wave antenna and the other uses varactors.

## 2. Analysis of 1-D periodic loading in transmission line.

Fig. 1 shows equivalent circuit model of the unit cell that contains load Z in series between two identical sections of transmission line. If the unit cell is symmetric, the propagation constant of the periodic structure is derived either by using ABCD parameters of the unit cell like (1), or directly calculated from the S-parameters of the unit cell like (2) [2].

$$\gamma = \alpha + j\beta = \frac{1}{d}\cosh^{-1}\left(\frac{A+D}{2}\right) = \frac{1}{d}\cosh^{-1}\left(\cos(kd) + \frac{jZ}{2Z_0}\sin(kd)\right) \tag{1}$$

$$\gamma = \alpha + j\beta = \frac{1}{d}\cosh^{-1}\left(\frac{1-S_{11}^2+S_{21}^2}{2S_{21}}\right) \tag{2}$$

In some structures, the dispersion diagram of infinite periodic case is obtained from (1) or (2). Fig. 2 shows the example of the dispersion diagram. In Fig. 2, The first passband appear between 4.4 GHz and 7.4 GHz, where the attenuation constant $\alpha$ is small. Inside the first passband nearby the stopband between 4.4GHz and 6.2GHz, there exists the band that the phase constant $\beta$ is smaller than free space wavenumber $k_0$.This region is so-called leaky region. In the leaky region, the radiation angle of the antenna is determined by the ratio of phase constant and free space wavenumber like

$$\cos\theta = \frac{\beta}{k_0} \tag{3}$$

$\theta$ is the angle from the axis parallel to the transmission line. Phase constant $\beta$ is changed not only by the change of frequency but also by the change of the loaded impedance Z. Therefore, the radiation angle of the antenna can be controlled by the change of frequency and the load impedance.

To excite the arrays without losing periodic properties, input and output port should be matched to the characteristic impedance of periodic wave. In here, this impedance is called Bloch impedance and is derived as

$$Z_B = Z_0 \frac{V_{n+1}}{I_{n+1}} = \frac{-BZ_0}{A - e^{\gamma d}} = \frac{\pm BZ_0}{\sqrt{A^2 - 1}} \tag{4}$$

So the impedances of input and output ports is matched to (4) with known impedance matching technique like quarter wavelength transformer.

## 3. Leaky wave antenna using DGS arrays

Fig. 3 shows the basic antenna structure of periodic DGS arrays. Wide defected areas and narrow connecting slot that operates like inductance and capacitance makes the variation of propagating waves. So the DGS element can be modelled equivalently to parallel L and C that is connected to the transmission line in series. To make more periodic perturbation, L and C is increased by enlarging the slot length. The parameters of equivalent circuit model are obtained from resonance and cut-off frequencies of single DGS cell [5]. The extracted parameter is 1.95nH and 0.41pF. The propagation constant $\gamma$ is derived from (1) and expressed as

$$\gamma = \frac{1}{d}\cosh^{-1}\left(\cos(kd) + \frac{\omega L}{2Z_0(1-\omega^2 LC)}\sin(kd)\right) \tag{5}$$

The dispersion diagram of periodic DGS arrays obtained from (5) is shown in Fig. 4 and the Bloch impedance is shown in Fig. 5. In there, the first passband is found from DC to 4.2GHz. In this band, the slow wave factor changes from 2.4 to 3.5. The second and third passbands exist at 6.5GHz-8GHz and 11GHz-17GHz. Inside the passband, the leaky regions, where $\beta / k_0$ is lower than 1, exist at 6.5GHz-7GHz and 12GHz-17GHz. However, the first leaky region (6.5GHz-7GHz) doesn't have sufficient frequency range and so the $\beta / k_0$ variation is not sufficient. On the other hand, the second leaky region (12GHz-17GHz) has a broad leaky region, a full varying range of $\beta / k_0$, and $60\,\Omega$~$70\,\Omega$ Bloch impedance. Therefore, second leaky region is chosen to the leaky wave antenna. 16 arrays are used and a quarter wavelength impedance matching technique is used at the start and end section of the antenna.

Fig. 6 shows the measured E-plane radiation patterns. Due to the backward wave properties in second leaky region, the leaky wave radiates in the opposite direction to the propagating direction of periodic wave. And the leaky wave radiates not only above the substrate but also below substrate because the DGS also operates as radiator.

The measured result of tilting angle is changed from 63° to 8° in backward direction as the frequency variation from 12.5GHz to 17GHz, and the averaged antenna gain is 10.5dBi including connector losses.

## 4. Beam steerable leaky wave antenna using varactors

If the equivalent circuit model has periodic capacitor load in transmission line, the propagation constant $\gamma$ is derived as

$$\gamma = \alpha + j\beta = \frac{1}{d}\cosh^{-1}\left(\cos(kd) + \frac{1}{2Z_0\omega C}\sin(kd)\right) \tag{6}$$

In the leaky region, the radiation angle of the antenna can be controlled by changing the capacitance in (6). So varactors are loaded in microstrip line in series with bias circuits to control the main beam direction. The structure of the proposed antenna is depicted in Fig. 7. The antenna is composed of array of 16 unit cells, and each unit cell has a varactor of MA46H120 between the microstrip line and an additional bias circuit. The total antenna also has impedance matching sections at both ends. The capacitance of the varactor changes from 0.29pF to 0.17pF when the reverse bias voltage is imposed from 5V to 12V.

Fig. 8(a) shows the measured *E*-plane radiation patterns at the fixed bias voltage of 12 V. As expected, the radiation angle of the antenna is turned from broadside to the end-fire according to the change of frequency. The measured *E*-plane radiation patterns at the fixed frequency of 4.2 GHz is shown in Fig. 8(b). The radiation angle is changed from 90° to 34° when the bias voltage is changed from 12 V to 5 V. The maximum gain of proposed antenna is 5.5dBi.

## 5. Conclusion

Two leaky wave antennas using periodic loads in microstrip line have been proposed. To design the antenna, the properties of microstrip line that has periodic load are analysed and discussed, and the leaky region and Bloch impedance are estimated. The proposed antenna using DGS element has broad operating frequency range (12GHz ~ 17GHz), backward beam direction and dual beams. Another proposed antenna using varactor has a wide beam steering performance of 56° at the fixed frequency. The calculated tilting angle and the estimation of leaky region have very good agreement with the measurement.

## 6. Acknowledgement

This work was supported in part by MIC & IITA of Korea through IT Leading R&D Support Project.

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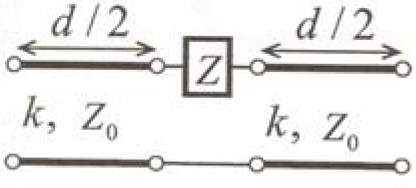


Fig. 1. Equivalent circuit model of the unit cell.

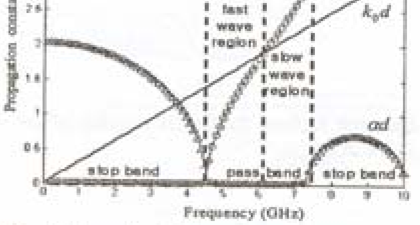


Fig. 2. Example of dispersion diagram.

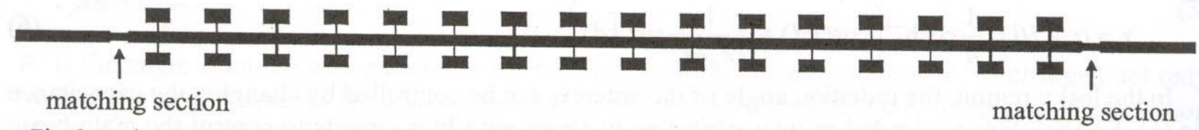

Fig. 3. Leaky wave antenna with the array of 16 DGS elements.

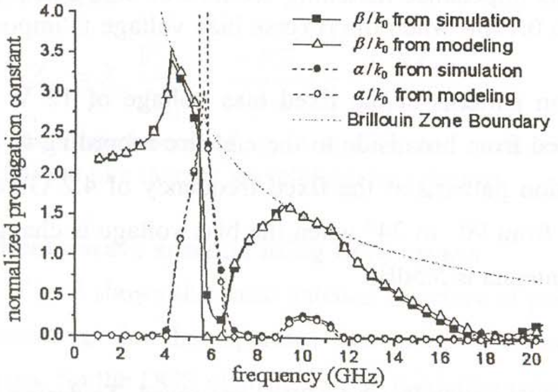

Fig. 4. Dispersion diagram of periodic DGS arrays.

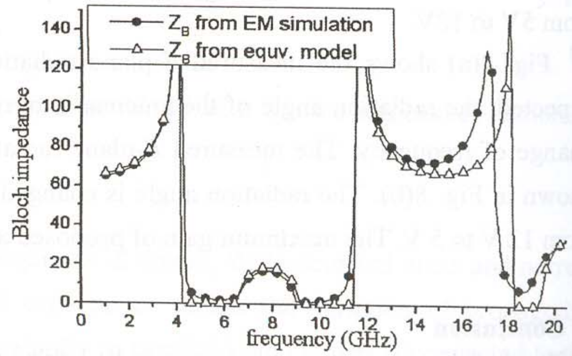

Fig. 5. Bloch impedance of periodic DGS arrays.

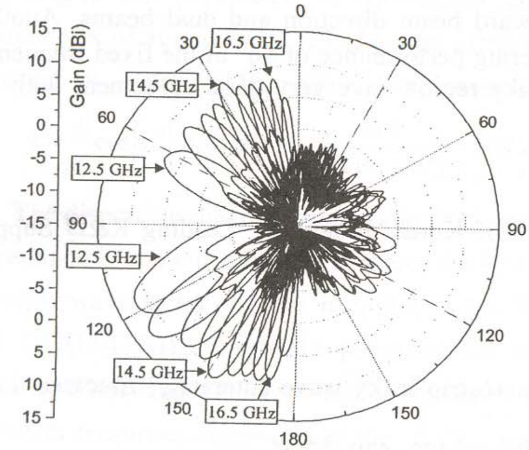

Fig. 6. Measured E-plane Radiation patterns of the antenna in Fig. 3.

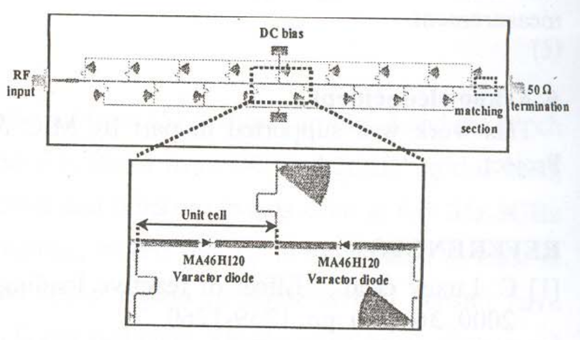

Fig. 7. Structure of the beam steerable leaky wave antenna that has periodic varactor loads and bias circuits.

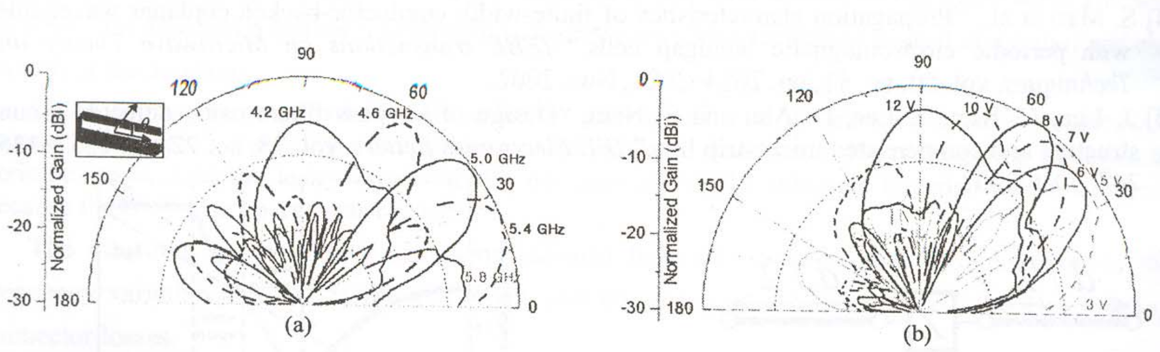

Fig. 8. Measured E-plane Radiation patterns of the antenna in Fig. 7. (a) at a fixed bias voltage of 12V and (b) at a fixed frequency of 4.2GHz.