Efficient Computation of the One-Dimensional Periodic Green's Function in a Multilayered Medium using the Ewald Method

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1. Introduction

An increase of the concern for 1-D periodic structure like leaky wave antenna often needs numerical full wave analysis to obtain accurate results in 1-D periodic structure. To use the Method of Moments (MoM) to 1-D periodic structure, fast and accurate methods for calculating 1-D periodic Green's function are often required. The Ewald method is one of the accelerating method for evaluating the periodic Green's function, and had already been applied to 2-D or 3-D periodic structures [1]-[4] and was recently applied to 1-D periodic line sources [5] and 1-D periodic dipole sources [6]. Here we apply the Ewald method with generalized pencil-of function (GPOF) method to accelerate the 1-D periodic Green's function in multilayered media, which are popularly used in circuits and antennas.

2. One-dimensional Periodic Green's function in a multilayered medium

When current sources, that is perpendicular to $\hat{z}$, are periodically distributed along x-axis with p_x periodicity in a multi-layered medium (see Fig. 1), the electric field based on mixed potential integral equation(MPIE) is expressed as

$$\mathbf{E}(\mathbf{r}) = -j\omega\mathbf{A}^P(\mathbf{r}) - \nabla\phi^P(\mathbf{r}) \tag{1}$$

where $\mathbf{A}^P$ is 1-D periodic magnetic vector potential and ϕ^P is scalar electric potential [7]. Due to the modal surface current $\mathbf{J}$ in the unit cell, $\mathbf{A}^P$ and ϕ^P are given by

$$\mathbf{A}^P(\mathbf{r}) = \int_S \underline{\mathbf{G}}_A^P(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')dS' \tag{2}$$

$$\phi^P(\mathbf{r}) = \int_S G_\phi^P(\mathbf{r},\mathbf{r}')[\nabla'\cdot\mathbf{J}(\mathbf{r}')]dS' \tag{3}$$

where $\underline{\mathbf{G}}_A^P$ and G_ϕ^P are the dyadic and scalar Green's functions for the 1-D periodic vector and scalar potentials $\mathbf{A}^P$ and ϕ^P. In a multilayered medium, the components of dyadic Greens function are simply expressed as $G_{A,xy}^P = G_{A,yx}^P = 0$ and $G_{A,xx}^P = G_{A,xx}^P = G_A^P$ thus only one component needs to be calculated. Using nonperiodic spectral-domain Green's function, 1-D periodic vector potential Green's function G_A^P and scalar electric potential G_ϕ^P are

$$G_{A,\phi}^P(X,Y) = \frac{1}{2\pi p_x}\sum_{n=-\infty}^{\infty} e^{-jk_{xn}X}\int_{-\infty}^{\infty}\tilde{G}_{A,\phi}(k_{xn},k_y)e^{-jk_yY}dk_y \tag{4}$$

where $X = x - x'$, $Y = y - y'$ and $k_{xn} = k_{x0} + 2\pi n/p_x$ ($n = 0$, ±1, ±2, ...). Nonperiodic spectral-domain Green's functions are easily obtained by the spectral-domain immitance method in [8] and are expressed as $\tilde{G}_A = \frac{1}{j\omega}Z_{TE}$, $\tilde{G}_\phi = \frac{Z_{TE} - Z_{TM}}{k_\rho^2}$.

To obtain 1-D periodic Green's functions numerically, the integration in a infinite region should be evaluated. However the integrand decays slowly (in a single dielectric layer with ground plane, the convergence rate is $1/k_\rho$), the evaluation of the integration becomes the dominant time consuming part. To avoid the time consuming integration, nonperiodic Green's functions are approximated in terms of complex exponential summation by GPOF method in [3], [10] and extracted so that the integration time can be significantly reduced by removing most of the integration. The approximated Green's functions generated by GPOF method are expressed as $\tilde{G}_A(k_\rho) \approx \sum_{i=1}^{I_A} c_i^A \frac{e^{s_i^A k_\rho}}{k_\rho}$, $\tilde{G}_\phi(k_\rho) \approx \sum_{i=1}^{I_\phi} c_i^\phi \frac{e^{s_i^\phi k_\rho}}{k_\rho}$. At $k_\rho > k_{\rho,\max}$, approximated Green's functions are almost same as original ones with the negligibly small errors. Thus by substituting approximated Green's functions for nonperiodic Green's functions in the $k_\rho > k_{\rho,\max}$ region, 1-D periodic Green's functions in (4) are converted into

$$G_{A,\phi}^P(X,Y) = \frac{1}{2\pi p_x} \sum_{n\in N} e^{-jk_{xn}X} \int_{-k_{y,\max}}^{k_{y,\max}} \left(\tilde{G}_{A,\phi} - \sum_{i=1}^{I_{A,\phi}} c_i^{A,\phi} \frac{e^{s_i^{A,\phi} k_{\rho n}}}{k_{\rho n}} \right) e^{-jk_y Y} dk_y + \frac{1}{2\pi} \sum_{i=1}^{I_{A,\phi}} c_i^{A,\phi} G_s(X,Y,s_i^{A,\phi}) \tag{5}$$

where $k_{y,\max} = \sqrt{k_{\rho,\max}{}^2 - \mathrm{Re}(k_{xn}{}^2)}$, $N = \{n \mid \sqrt{\mathrm{Re}(k_{xn}{}^2)} < k_{\rho,\max}\}$, $k_{\rho n} = \sqrt{k_{xn}{}^2 + k_y{}^2}$ and

$$G_s(X,Y,s_i^{A,\phi}) = \frac{1}{p_x} \sum_{n=-\infty}^{\infty} e^{-jk_{xn}X} \int_{-\infty}^{\infty} \frac{e^{s_i^{A,\phi} k_{\rho n}}}{k_{\rho n}} e^{-jk_y Y} dk_y \tag{6}$$

The first term of right side in (5) needs only a few integrations over finite small intervals so that the evaluation time for the integrations is significantly reduced. The integration paths should be carefully determined because of Branch points, surface wave poles on the proper sheet and leaky wave poles on the improper sheet of the complex k_y plane. The guide lines are given in [7] and [11]

Applying Poisson summation transform and some efforts, the second term of right side in (5) is converted to the scalar potential generated by the 1-D periodic static sources which are linearly phased along $\hat{x}$. Thus using Ewald method in [6] the summation of (6) can be accelerated by dividing into two parts, spatial domain and spectral domain.

$$G_s(X,Y,s_i^{A,\phi}) = \sum_{n=-\infty}^{\infty} e^{-jk_{x0}np_x} \frac{1}{R_{n,s_i^{A,\phi}}} = G_{spatial} + G_{spectral} \tag{7}$$

where $R_{n,s_i^{A,\phi}} = \sqrt{(X - np_x)^2 + Y^2 + (s_i^{A,\phi})^2}$.

$$G_{spatial}(X,Y,s_i^{A,\phi}) = \sum_{n=-\infty}^{\infty} e^{-jk_{x0}np_x} \frac{erfc(R_{n,s_i^{A,\phi}}/2E)}{R_{n,s_i^{A,\phi}}} \tag{8}$$

$$G_{spectral}(X,Y,s_i^{A,\phi}) = \frac{1}{p_x} \sum_{n=-\infty}^{\infty} e^{-jk_{xn}X} \left(\sum_{q=0}^{\infty} \frac{\left[-\{Y^2 + (s_i^{A,\phi})^2\}/(4E^2) \right]^q}{q!} \mathrm{E}_{q+1}(k_{xn}{}^2 E^2) \right) \tag{9}$$

Thus the second term of right side is expressed as closed form and numerically only a few terms are needed to calculate the summation in (8) and (9) due to the fast convergence properties. In (9), the evaluation of exponential integral function $\mathrm{E}_{q+1}(z)$ is needed. Fortunately, higher order exponential integrals are obtained by the recurrence relation in [9, Sec. 5.1] so that only $\mathrm{E}_1(z)$ needs to be evaluated numerically.

When the observation point is on the source point, source singularities are occurred in (4) and (5), and singular contribution should be included in performing the MoM. In (4), the source singularity is easily extracted, because (8) contains source singularity 1/R when $s_i^{A,\phi} = 0$.

3. Numerical Results

A typical 1-D periodic structure in a multilayered medium is microstrip line with periodic perturbation with ground plane. Thus a single dielectric layer with a ground plane is considered and tested in here. The dielectric layer has relative permittivity ε_r, relative permeability $\mu_r = 1$, and thickness h. Fig. 2 shows the validity of GPOF approximation, the approximated Green's function has the error that is lower than 10^{-4} compared to the exact Green's function in the $k_\rho > k_{\rho,\max}$. In here, $k_{\rho,\max} = 2.5k_0$ is chosen. Fig. 3 shows the convergence of Ewald method. Until satisfying the convergence of the summation, only a few terms are needed. Fig. 4 shows the calculated 1-D periodic Greens' functions and the comparison with reference results in [7]. The total used time for evaluating Green's function by our method at 25 observation points is 8.0 seconds which is slightly larger than 7.5 seconds by the method in [7]. In here, about 85% time is used for evaluating $\mathrm{E}_1(z)$ numerically that means the major time spent part in our method is numerical evaluation of $\mathrm{E}_1(z)$. If the approximations in [9, Sec. 5.1.53-56] are used to get $\mathrm{E}_1(z)$, the total used time is reduced to 1.2 seconds that is more faster than that in [7] but if the input argument z has a large imaginary part or a negative real part, the approximations in [9, Sec. 5.1.53-56] are not useful. Nevertheless our method is 1000 or more times faster than direct calculation of (4).

4. Conclusion

The 1-D periodic Green's function in a multilayered medium is accelerated using the GPOF method and the Ewald method, and it is verified in a single dielectric layer with a ground plane. Although this method has the time consuming problem in evaluating exponential integral functions, still it guarantees fast computation time. And any other efforts are not needed to extract source singularities because 1/R singularities are already shown in the closed form expressions.

5. Acknowledgement

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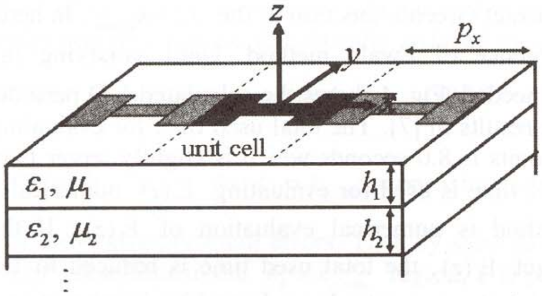

Fig. 1. Example of 1-D periodic planar structure in a multilayered medium with relevant physical and geometrical parameters.

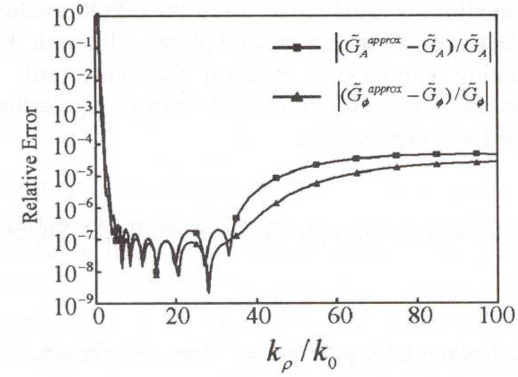

Fig. 2. Relative errors of nonperiodic spectral-domain Green's functions approximated by GPOF method. Spectral-domain Green's functions in single dielectric layer with a ground plane, which has $\varepsilon_r = 2.2$, $\mu_r = 1$, and $h = 1.5mm$, are approximated and compared to original ones at 20GHz.

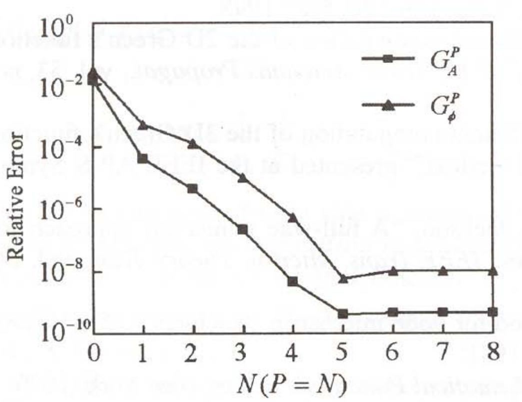

Fig. 3. Relative errors versus number of terms $N(P = N)$ in the Ewald sum in (7) with (8) and (9) calculated at $\mathbf{r} = (x, y, z) = (1mm, 1.5mm, 0mm)$ with the period of $p_x = 2mm$, the frequency of 20GHz and same physical parameters of Fig. 2.

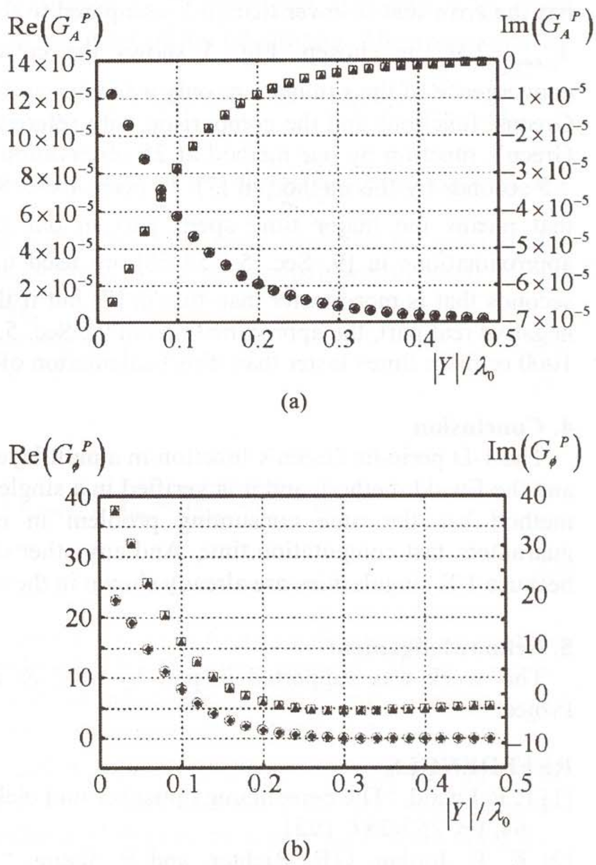

Fig. 4. Comparison between 1-D periodic Green's functions obtained using the Ewald Method and the method in [7] at 20GHz. (a) G_A^P and (b) G_ϕ^P are plotted as functions of the normalized absolute value of the distance $|Y|/\lambda_0$ between source and observation points. The background structure is the same as in Fig. 1 and Fig. 2 and normalized phase constant of the fundamental harmonic is $\beta_{x0}/k_0 = 1.25$ and normalized attenuation constant is $\alpha_x/k_0 = 0.05$.