Analysis of Wireless Power Transfer Using Spherical Modes

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ABSTRACT: To investigate the wireless power transfer, we need to analyze the antenna coupling. We derive a Z-parameter between two antennas using spherical modes and the addition theorem. We present formulas that calculate the maximum power transfer efficiency and the optimum load impedance for the antennas generating arbitrary modes. To simplify the formula, we assume that the antennas are canonical minimum scattering antennas. We find from the formula presented in this paper that the power transfer efficiency increases as the mode number and the radiation efficiency increase.

INTRODUCTION

Recently, wireless power transfer using near-field is receiving much attention and is being studied extensively. In order to investigate the characteristics of wireless power transfer, we need to analyze the antenna mutual coupling, as the power transfer takes place between antennas through the coupling phenomenon. The coupling between antennas was described in terms of the Z-parameter and it was shown that the transfer efficiency is a function of distance and the radiation efficiency of the antennas generating the fundamental modes only [1]. In the present paper, we exploit the Z-parameter derived by Wasylkiwskyj and Kahn[2] using the addition theorem to derive a formula that calculates the maximum power transfer efficiency and the optimum load impedance for near-field wireless power transfer system using antennas generating higher-order modes. To simplify the formula, we assume antennas to be canonical minimum scattering (CMS) antennas, which is a valid assumption for the types of small antennas used for wireless power transfers. We find that an efficient wireless power transfer can be achieved using a higher order spherical mode.

Z-PARAMETER BETWEEN TWO ANTENNAS

In order to derive the Z-parameter between two antennas' port, we first express an antenna as a scattering matrix [3].

$$\begin{bmatrix} w \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} \Gamma & \mathbf{R} \\ \mathbf{T} & \mathbf{S} \end{bmatrix} \begin{bmatrix} v \\ \mathbf{a} \end{bmatrix} \tag{1}$$

The radiation, receiving and scattering properties of an antenna are contained in this matrix equation. Let antenna 1 be on the origin of coordinate 1 (x_1, y_1, z_1) and antenna 2 be on the origin of coordinate 3(x_3, y_3, z_3), as shown in Fig. 1. The coordinate 2 (x_2, y_2, z_2) is obtained by translating coordinate 1 by $\mathbf{r}$ and coordinate 3 is obtained by rotating coordinate 2. It is assumed that two spheres enclosing each antenna do not overlap. The coupling of two antennas can be considered as the cascading of a transmitting antenna network, a space network, and a receiving antenna network, as shown in Fig. 2. Here, the antenna networks are expressed as the scattering matrix and the space network is expressed as the Z-parameter. In Fig.2, the left side ports in the space network denote the mode ports for coordinate 1 and the right side ports are the mode ports of coordinate 3. In order to derive the Z-parameter of the space network, we need to convert the mode coefficients in coordinate 1 into the mode coefficients in coordinate 3. First, the mode coefficients in coordinate 1 are converted into the mode coefficients in coordinate 2 by means of the addition theorem [4]. Second, the mode coefficients in coordinate 2 are converted into the mode coefficients in coordinate 3 by means of the formula in [4, Appendix 3]. Let $\mathbf{a}^{(i)}$ and $\mathbf{b}^{(i)}$ be mode coefficients of the incoming and outgoing waves in coordinate i (i=1,2,3), respectively. Then, in the sphere whose origin is that of coordinate 3 with a radius of $|\mathbf{r}|$, the relationship between $\mathbf{a}^{(3)}$ (or $\mathbf{a}^{(1)}$) and $\mathbf{b}^{(1)}$ (or $\mathbf{b}^{(3)}$) are expressed by the matrix equation

$$\mathbf{a}^{(3)} = \mathbf{b}^{(3)} = \mathbf{G}^{+}\mathbf{b}^{(1)} \tag{2}$$

$$\mathbf{a}^{(1)} = \mathbf{b}^{(1)} = \mathbf{G}^{-}\mathbf{b}^{(3)} \tag{3}$$

The matrix $\mathbf{G}^{+}$ and $\mathbf{G}^{-}$ are defined in [4]. The Z-parameter of the space network is

$$\mathbf{Z}^{S} = \left[\begin{array}{c|c} \mathbf{I} & \mathbf{G}^{-} \\ \hline \mathbf{G}^{+} & \mathbf{I} \end{array}\right] \tag{4}$$

where **I** is a unit matrix[4]. Suppose that the scattering matrices of antennas 1 and 2 are, respectively,

$$\mathbf{S}^{(1)} = \left[\begin{array}{c|c} \Gamma_1 & \mathbf{R}_1 \\ \hline \mathbf{T}_1 & \mathbf{S}_1 \end{array}\right], \quad \mathbf{S}^{(2)} = \left[\begin{array}{c|c} \Gamma_2 & \mathbf{R}_2 \\ \hline \mathbf{T}_2 & \mathbf{S}_2 \end{array}\right]. \tag{5}$$

Let the characteristic impedance of the local port of antenna 1 and 2 be, respectively, Z_{01} and Z_{02}. The Z-parameter between the two antennas' local ports is

$$\begin{aligned} Z_{11} = Z_{01}\frac{1+\Gamma_1}{1-\Gamma_1} - \frac{Z_{01}}{2(1-\Gamma_1)^2}\mathbf{R}_1\mathbf{G}^{-}\left(\mathbf{I}-\mathbf{S}_O^{(2)}\right) \\ \cdot\mathbf{G}^{+}\left[\mathbf{I}-\frac{1}{4}\left(\mathbf{I}-\mathbf{S}_O^{(1)}\right)\mathbf{G}^{-}\left(\mathbf{I}-\mathbf{S}_O^{(2)}\right)\mathbf{G}^{+}\right]^{-1}\mathbf{T}_1 \end{aligned} \tag{6}$$

$$\begin{aligned} Z_{21} = \frac{\sqrt{Z_{01}Z_{02}}}{(1-\Gamma_1)(1-\Gamma_2)}\mathbf{R}_2 \\ \cdot\mathbf{G}^{+}\left[\mathbf{I}-\frac{1}{4}\left(\mathbf{I}-\mathbf{S}_O^{(1)}\right)\mathbf{G}^{-}\left(\mathbf{I}-\mathbf{S}_O^{(2)}\right)\mathbf{G}^{+}\right]^{-1}\mathbf{T}_1 \end{aligned} \tag{7}$$

$$\begin{aligned} Z_{12} = \frac{\sqrt{Z_{01}Z_{02}}}{(1-\Gamma_1)(1-\Gamma_2)}\mathbf{R}_1 \\ \cdot\mathbf{G}^{-}\left[\mathbf{I}-\frac{1}{4}\left(\mathbf{I}-\mathbf{S}_O^{(2)}\right)\mathbf{G}^{+}\left(\mathbf{I}-\mathbf{S}_O^{(1)}\right)\mathbf{G}^{-}\right]^{-1}\mathbf{T}_2 \end{aligned} \tag{8}$$

$$\begin{aligned} Z_{22} = Z_{02}\frac{1+\Gamma_2}{1-\Gamma_2} - \frac{Z_{02}}{2(1-\Gamma_2)^2}\mathbf{R}_2\mathbf{G}^{+}\left(\mathbf{I}-\mathbf{S}_O^{(1)}\right) \\ \cdot\mathbf{G}^{-}\left[\mathbf{I}-\frac{1}{4}\left(\mathbf{I}-\mathbf{S}_O^{(2)}\right)\mathbf{G}^{+}\left(\mathbf{I}-\mathbf{S}_O^{(1)}\right)\mathbf{G}^{-}\right]^{-1}\mathbf{T}_2 \end{aligned} \tag{9}$$

where

$$\mathbf{S}_O^{(1)} = \frac{1}{1-\Gamma_1}\mathbf{T}_1\mathbf{R}_1 + \mathbf{S}_1 \tag{10}$$

$$\mathbf{S}_O^{(2)} = \frac{1}{1-\Gamma_2}\mathbf{T}_2\mathbf{R}_2 + \mathbf{S}_2 . \tag{11}$$

We must know the full scattering matrix of an antenna to calculate a Z-parameter. In order to determine the scattering matrix, we assume that antennas are a canonical minimum scattering (CMS) antenna, which does not scatter the electromagnetic fields when its local ports are open-circuited. The approximation is thought to be valid for the antennas that are small compared with wavelength. The scattering matrix of a CMS antenna has been derived by Kahn and Kurss [5]. The **S** of a matched CMS antenna is

$$\mathbf{S} = \mathbf{I} - \mathbf{T}\mathbf{R} . \tag{12}$$

MAXIMUM POWER TRANSFER EFFICIENCY AND OPTIMUM LOAD IMPEDANCE

If the Z-parameter is given, the load impedance to which power is transferred maximally and the maximum power transfer efficiency can be calculated. Maximum power transfer occurs when the load impedance is the following equation.

$$\mathrm{Re}(Z_L) = \sqrt{\mathrm{Re}(Z_{22})^2 - \frac{\mathrm{Re}(Z_{22})}{\mathrm{Re}(Z_{11})}\mathrm{Re}(Z_{12}Z_{21}) - \frac{\mathrm{Im}(Z_{12}Z_{21})^2}{4\,\mathrm{Re}(Z_{11})^2}} \tag{13}$$

$$\mathrm{Im}(Z_L) = \frac{\mathrm{Im}(Z_{12}Z_{21})}{2\,\mathrm{Re}(Z_{11})} - \mathrm{Im}(Z_{22}) \tag{14}$$

When antennas are reciprocal, maximum power transfer efficiency is

$$PTE^{max} = \frac{|X|^2}{2 - \mathrm{Re}\left(X^2\right) + \sqrt{4 - 4\,\mathrm{Re}\left(X^2\right) - \mathrm{Im}\left(X^2\right)^2}} \tag{15}$$

where $X = Z_{21} / \sqrt{\mathrm{Re}(Z_{11})\,\mathrm{Re}(Z_{22})}$.

WIRELESS POWER TRANSFER BETWEEN SINGLE MODE ANTENNA

As an application of the formula presented above, we try to find if there is any spherical mode that is more efficient for wireless power transfer than the fundamental mode. In order to simplify the formula, we assume antennas to be reciprocal matched MS antennas that generate only one TM_{0n} (or TE_{0n}) mode. Assume that ϕ_0, θ_0, and χ_0 are 0. Let the radiation efficiency of antenna 1 be η_{rad1} and the radiation efficiency of antenna 2 be η_{rad2}. In this case, the X in (15) becomes

$$X = \sqrt{\eta_{rad1}\eta_{rad2}}\, A^{(4)}_{0\nu,0n}\left(r,\theta,\phi\right) \tag{16}$$

where $A^{(4)}_{0\nu,0n}\left(r,\theta,\phi\right)$ is the function in the addition theorem [3]. Fig. 3 shows the maximum power transfer efficiencies between the TM_{0n} mode antenna and the $TM_{0\nu}$ mode antenna when the radiation efficiencies are 1. In Fig. 3, the maximum power transfer efficiency increases as the mode number increases. This phenomenon occurs because the higher the mode number is, the larger the value of Q becomes. Because the high Q antenna is efficient for a wireless power transfer scheme [6], the higher order mode antenna is efficient. The maximum power transfer efficiency increases as the radiation efficiency increases because the larger the radiation efficiency is the larger the magnitude of X becomes. Fig. 4 shows the maximum power transfer efficiencies between two TM_{01} (TE_{01}) mode antennas and between two TM_{02} (TE_{02}) mode antennas for several radiation efficiencies.

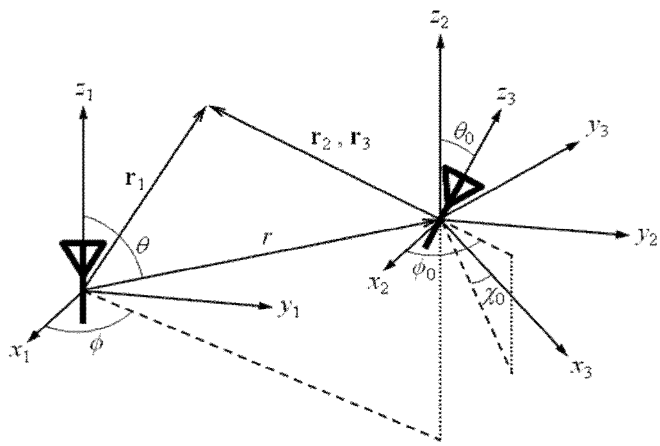

Fig. 1. Coordinate systems and antennas

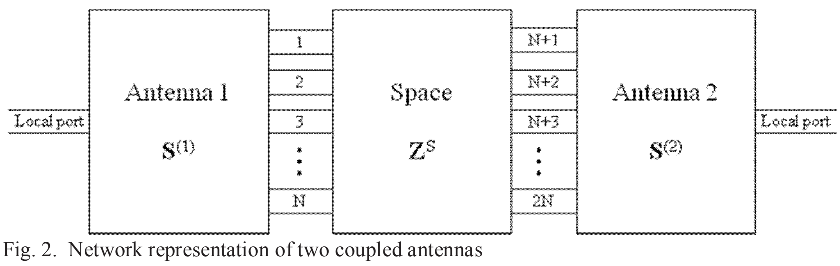

Fig. 2. Network representation of two coupled antennas

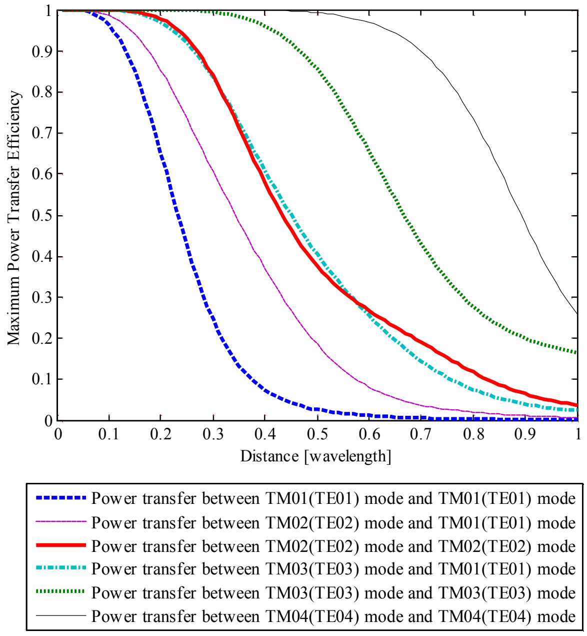

Fig. 3. Maximum power transfer efficiencies between the TM_{0n} (TE_{0n}) mode antenna and the TM_{0v} (TE_{0v}) mode antenna when $\eta_{rad} = 1$, $\theta = 0$, $\phi = 0$

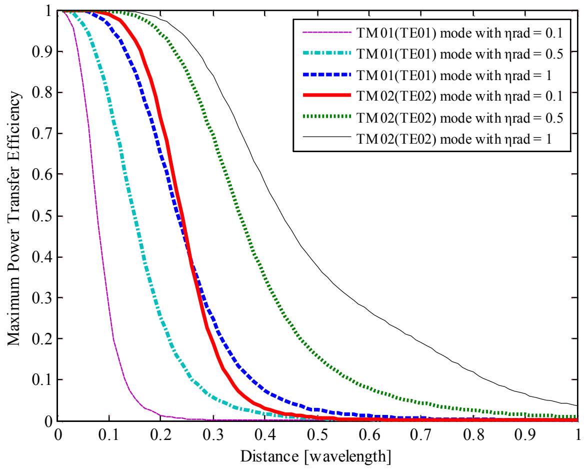

Fig. 4. Maximum power transfer efficiencies between antennas with different radiation efficiencies, $\theta = 0$, $\phi = 0$

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