Protecting the Method of Auxiliary Sources (MAS) Solutions From the Interior Resonance Problem

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***Abstract*—The method of auxiliary sources (MAS) has been used for the numerical solution of a variety of electromagnetic problems successfully. Since MAS can be considered as a modified or generalized version of the standard surface integral equation technique (SIE), it has the interior resonance problem, but the remedies to this problem have not been investigated yet. In this letter, it is shown that the interior resonance problem in applying MAS can be completely removed using combined-source solution (CSS).**

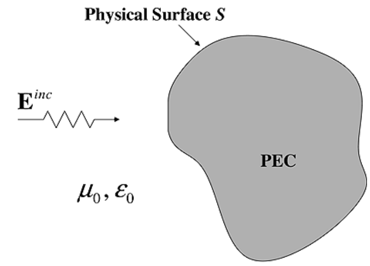
***Index Terms*—Combined-source solution (CSS), interior resonance, method of auxiliary sources (MAS).**

I. Introduction

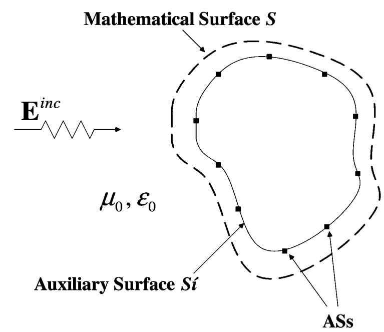
THE method of auxiliary sources (MAS) is a highly promising and versatile numerical technique to solve the boundary value problems arising in electromagnetic analysis [1]. MAS was introduced, named, and developed by a research group in Georgia (part of the former Soviet Union) [2]. This method has some different names such as "Generalized Formulations" [3] since other research groups elsewhere in the world have independently developed the same method. Reference [1] reviews various aspects of MAS including historical perspectives, fundamentals of the method, and current status of the research activities. It also has an extensive list of references; hence interested readers are referred to that article.

For unfamiliar readers with this method, the basic procedure of MAS is briefly explained here. We choose the simplest two-dimensional (2–D) scattering analysis case for the purpose of presentation.

Fig. 1(a) shows the problem geometry. There is a perfect electrically conducting (PEC) scatterer and it is illuminated by an incident wave. In the standard surface integral equation technique (SIE), the unknown currents (chosen bases with unknown coefficients) are distributed on the scatterer surface by the equivalence principle. Then the unknown coefficients are obtained using the method of moments (MoM) [4]. In contrast to this, when applying MAS, discrete auxiliary sources (ASs) are located on an auxiliary surface enclosed by the physical scatterer surface as shown in Fig. 1(b). This auxiliary surface is usually conformal to the physical scatterer surface, but not necessarily is. Then the problem is solved imposing the boundary condition that the tangential electric field vanishes at the physical scatterer surface in the same way as the standard SIE. When the solution



(a)



(b)

Fig. 1. (a) PEC scatterer illuminated by an incident wave. (b) MAS model equivalent to the situation in Fig. 1(a).

is completed, the coefficients of ASs are obtained, and then the needed physical quantities such as the surface currents and the radar cross section (RCS) can be calculated using these values. The coefficients of ASs themselves do not have physical significance.

In summary, MAS adopts the discrete sources located in the inside of the physical scatterer and distant from the surface. Therefore the numerical integrations and singularity extractions are not needed, hence the computational procedure is more simple and efficient than that of the standard SIE.

II. Problem Statement

As stated above, MAS is a modified or generalized version of the standard SIE. Therefore, it has the interior resonance problem. Unlike the standard SIE, resonant frequencies where the solutions are corrupted are determined by the auxiliary surface where ASs reside, not by the physical scatterer surface itself [3]. In applying MAS, the extent of the solution corruption by the interior resonance is subject to the location of the ASs, and in some cases the solution corruptions are negligible. There is a paper which points out this feature and states that the interior resonance problems are avoided by MAS [5], but its conclusion is a hasty generalization. Their reasoning makes sense but is far

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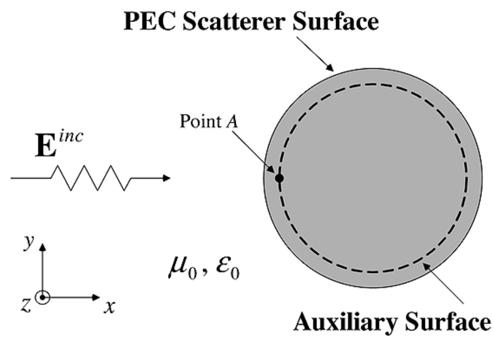


Fig. 2. 2-D circular cylinder illuminated by a TM-z polarized plane wave.

from being complete. To prove this statement, we work out the same problem as in [5] and present refutations.

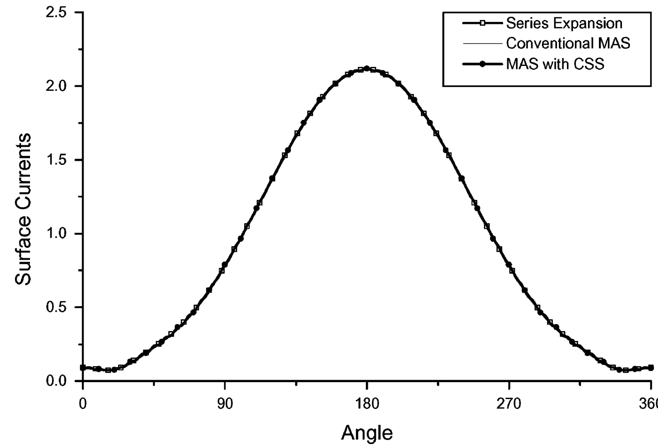
The problem under consideration is a scattering by a PEC cylinder of radius R illuminated by a TM plane wave as depicted in Fig. 2. An auxiliary surface S_i is selected to be a concentric, hence conformal cylinder of radius $R_i = 0.8R$. In [5] the surface currents on the cylinder are calculated for different values of k_0R (k_0: wavenumber of the free space), and the results show that the surface currents from MAS remain accurate for $k_0R = 3.832$, which corresponds to the resonant frequency of the cylinder [6, Table 5.2]. For the same frequency, the surface currents from the standard SIE are corrupted by the interior resonance problem. And that paper also states that although the frequency corresponding to $k_0R = 6.420$ is a resonant frequency of S_i (because $k_0R_i = 5.136$), the results based on MAS remain accurate. Our results are quite the same in these cases.

However, if we place the auxiliary surface a little bit closer to the scatterer surface at $R_i = 0.85R$, the situation is changed dramatically. We compare the results with those from the series expansion method for $k_0R = 2.4048$ and $k_0R = 2.8292$, which corresponds to $k_0R_i = 2.4048$. For the case of $k_0R = 2.4048$ shown in Fig. 3(a), the results remain accurate since the resonant frequencies in applying MAS are determined by the auxiliary surface. But for the case of $k_0R = 2.8292$ shown in Fig. 3(b), the results are severely corrupted by the interior resonance, which cannot be explained following the reasoning in [5]. If we plot the absolute values of the coefficients of ASs versus k_0R, then the resonant peaks are clearly shown as in Fig. 4(a) (for the case of $R_i = 0.8R$) and Fig. 4(b) (for the case of $R_i = 0.85R$). Note that the locations of the resonant peaks in two figures are different since the auxiliary surfaces determine the resonant frequencies. Even though the surface currents obtained from these auxiliary sources can be quite accurate in some cases, there is no guarantee that they will be so.

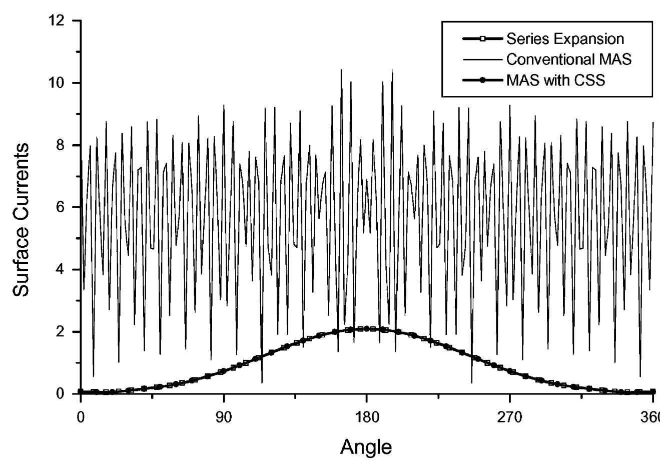
Unfortunately, little attention has been given to dealing with the interior resonance problem in applying MAS, and we couldn't find any article that suggests a remedy. But this problem should be resolved for MAS to be more robust.

III. Combined-Source Solution

In the standard SIE, there have been many efforts to deal with the interior resonance problem, and the most robust and widely used scheme is the combined field integral equation (CFIE) [7].



(a)

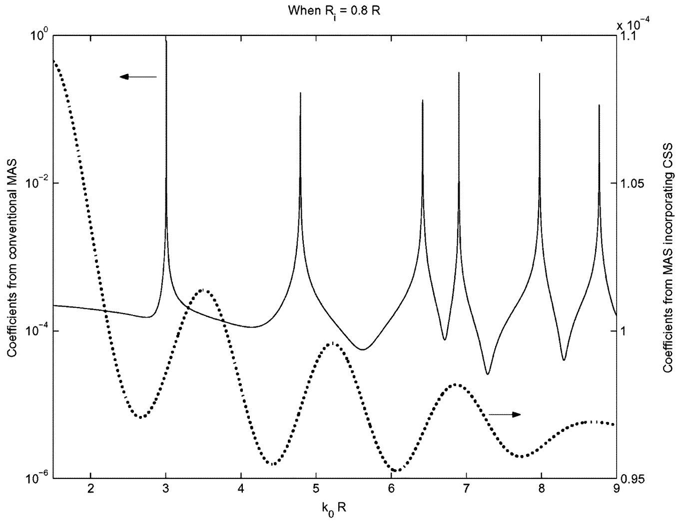


(b)

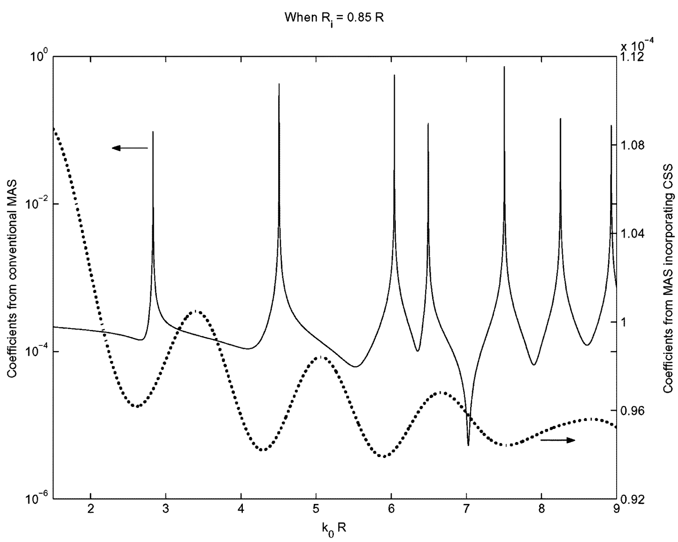
Fig. 3. Surface currents calculated from series expansion, conventional MAS, and MAS incorporating CSS for the case of (a) $k_0R = 2.4048$ and (b) $k_0R = 2.8292$.

CFIE is a linear combination of the electric field integral equation (EFIE) and the magnetic field integral equation (MFIE) formulated using unknown electric current sources only for PEC scattering analysis. Unfortunately, MFIE is not directly applicable to MAS since the surface where the unknown sources reside and the surface where the boundary condition is imposed are different. Therefore, CFIE is not applicable to MAS.

There is a dual approach to CFIE, although not so widely used as CFIE. It is the combined-source solution (CSS) [8]. CSS is formulated using both the electric current source $\mathbf{J}$ and the magnetic current source $\mathbf{M}$ and these are related by $\mathbf{M} = \eta\hat{n} \times \mathbf{J}$, where η is the intrinsic impedance of the free space and $\hat{n}$ is the outward normal to the surface where the sources reside. And the boundary condition is imposed using tangential electric field only. Hence CSS is applicable to MAS. The main reason why CSS is not so widely used may be the fact that the electric current $\mathbf{J}$ obtained using CSS has no physical significance itself. To obtain the electric current $\mathbf{J}^c$ which does have physical significance, an additional calculation is needed using both $\mathbf{J}$ and $\mathbf{M}$. However, this feature of CSS does not seem to be a drawback in applying CSS to MAS since in using MAS the coefficients of



(a)



(b)

Fig. 4. Absolute values of the coefficients of the AS at point A in Fig. 2 calculated from conventional MAS (log scale, solid line) and MAS incorporating CSS (linear scale, dotted line) for the case of (a) $R_i = 0.8R$ and (b) $R_i = 0.85R$.

ASs are obtained first and then needed quantities are calculated using these values, naturally.

Results from MAS incorporating CSS are shown in Fig. 3(a) and (b) and Fig. 4(a) and (b). Fig. 3(a) and (b) show the surface currents and they agree very well with those from the series expansion method unlike those from conventional MAS. If we plot the absolute values of the coefficients of ASs versus k_0R, we can assure that there are no resonant peaks in the solutions from MAS incorporating CSS and they are clearly shown in Fig. 4(a) and (b). The results show that the solutions from MAS incorporating CSS are not affected by the interior resonance problem at all.

IV. Conclusion

In this letter, we have shown that MAS incorporating CSS completely removes the interior resonance problem. With this approach, MAS becomes more robust and hence the applicability of MAS can be expanded a lot.

Numerical examples are presented for 2-D scattering analysis case, but the extension to three-dimensional case is straightforward.

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