Analysis of scattering from a 3-D doubly periodic cavity array using iterative FEM

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Abstract : In this paper, we present the formulation of the iterative finite element method for the analysis of scattering from a three-dimensional doubly periodic cavity array. The single unit cell is discretized with the triangular prismatic volume elements and the electric field intensity is expanded with the edge-based basis functions. The periodicity is incorporated in the Green's function and its computation is accelerated by the Ewald transformation. By virtue of the finite element method, the proposed scheme is applicable to periodic arrays comprised of cavities having arbitrary shape and filled with inhomogeneous dielectrics. In addition, the iterative nature using radiation-type boundary condition makes this scheme free from the interior resonance problem. To validate the proposed method, a numerical example is presented.

Ⅰ. Introduction

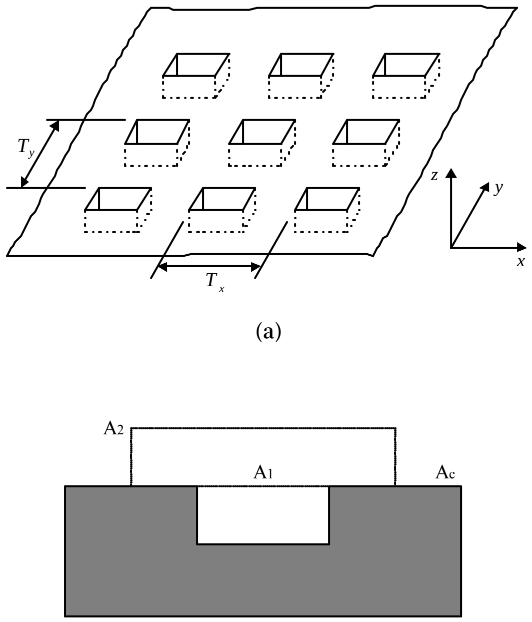
When solving an open region problem such as radiation or scattering via finite element method (FEM), it is common practice to use the hybrid method such as the finite element boundary integral method (FEBIM) or incorporate various kinds of absorbing boundary conditions (ABC) [1]. Although these methods are quite versatile and have given successful results in a variety of literature, yet they have inherent shortcomings. As an alternative to these methods, the iterative FEM was proposed which preserves the symmetry and the bandedness of the FEM system matrix [2-3]. But, early efforts using the iterative FEM incorporated the Dirichlet type boundary conditions and are prone to the well-known interior resonance problem. In order to overcome this drawback, new schemes which adopt the radiation-type (mixed) boundary conditions are devised and successfully applied [4-6].

In this paper, we extend the aforementioned iterative FEM for scattering by infinite cavity arrays. Since the Green's function for the doubly periodic structure is needed in the iterative FEM procedure and it is necessary to accelerate the computation of the Green's function, the Ewald transformation method is adopted.

Ⅱ. Formulation

As an example of the proposed method, the scattering by a three-dimensional doubly periodic rectangular cavity array recessed in a ground plane is analyzed. Only a single unit cell is discretized using the triangular prismatic volume elements and the electric field intensity is expanded with the edge-based basis functions. Fig. 1 shows the doubly periodic structure to be analyzed and the unit cell with a fictitious surface on which the boundary fields will be updated. It is noticeable that the boundary surface A2 in Fig. 1(b) can be placed quite close to the aperture surface A1. As mentioned above, the radiation type boundary condition should be applied on A2 in order to avoid the interior resonance-like phenomena. On the fictitious boundary A2, the boundary condition is imposed as follows.

$$\hat{n}\times\nabla\times\mathbf{E} + jk_0\hat{n}\times\hat{n}\times\mathbf{E} = \mathbf{U} \qquad (1)$$



(a)

(b)

Fig. 1. Problem geometry (a) periodic structure: T_x and T_y denote the periodicities of the periodic structure in the x and y direction, respectively. (b) single unit cell: A1 is the aperture surface where the equivalence principle is applied. A2 is the fictitious boundary where the radiation-type boundary condition is imposed. Ac is the PEC surface.

The left side of this equation has the form of Sommerfeld radiation condition. Hence in the far field region, the right hand side should be zero. However, in the near field region, this is simply not so. The key idea of the iterative FEM is based on the update of this residual term. With this mixed boundary condition, the functional is given as follows [1].

$$F(\mathbf{E}) = \frac{1}{2}\int_V \left\{ \frac{1}{\mu_r} (\nabla\times\mathbf{E})\cdot(\nabla\times\mathbf{E}) - k_0^2 \varepsilon_r \mathbf{E}\cdot\mathbf{E} \right\} dv + \frac{j k_0}{2}\int_{A_2} (\hat{n}\times\mathbf{E})\cdot(\hat{n}\times\mathbf{E})\, ds + \int_{A_2} \mathbf{E}\cdot\mathbf{U}\, ds \tag{2}$$

Initially, $\mathbf{U}$ is calculated from the equation (1) with an assumption that $\mathbf{E}$ in the equation (1) is the same as the incident electric field $\mathbf{E}_{\text{inc}}$. According to the typical FEM procedure, with the functional in the equation (2) minimized, the electric fields can be determined everywhere. From these calculated fields, the equivalent magnetic current source on the aperture surface A1 is introduced using the equivalence theorem. Then the scattered fields on the fictitious boundary due to these equivalent magnetic sources are calculated using the Green's function.

With the help of the image theory, the Green's function needed is that for doubly periodic structure residing in free space and is given by [7]

$$G_p(\mathbf{r},\mathbf{r}') = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} e^{-j\mathbf{k}_{t00}\cdot\boldsymbol{\rho}_{mn}} \frac{e^{-jk_0 R_{mn}}}{4\pi R_{mn}} \tag{3}$$

, where $R_{mn} = \left|\mathbf{r}-\mathbf{r}'-\boldsymbol{\rho}_{mn}\right|$, $\boldsymbol{\rho}_{mn} = \hat{x} m T_x + \hat{y} n T_y$ and $\mathbf{k}_{t00} = -k_0(\hat{x}\sin\theta_i\cos\phi_i + \hat{y}\sin\theta_i\sin\phi_i)$

This Green's function can also be expressed as the summation form in the spectral domain as

$$G_p(\mathbf{r},\mathbf{r}') = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{e^{-j\mathbf{k}_{tmn}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')}}{2jT_xT_yk_{zmn}} e^{-jk_{zmn}|z-z'|} \tag{4}$$

, where $\mathbf{k}_{tmn} = \mathbf{k}_{t00} + 2\pi(\hat{x}\, m/T_x + \hat{y}\, n/T_y)$ and

$$k_{zmn} = \sqrt{k_0^2 - \mathbf{k}_{tmn}\cdot\mathbf{k}_{tmn}}$$

Because neither form of the Green's function converges fast enough, the computation process is accelerated using the Ewald transformation and the result is given by [7]

$$G_p(\mathbf{r},\mathbf{r}') = G_{p1}(\mathbf{r},\mathbf{r}') + G_{p2}(\mathbf{r},\mathbf{r}') \tag{5}$$

, where

$$G_{p1}(\mathbf{r},\mathbf{r}') = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{e^{-j\mathbf{k}_{tmn}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')}}{4jT_xT_yk_{zmn}} \times\left[e^{-jk_{zmn}|z-z'|} erfc\left(\frac{jk_{zmn}}{2E} - |z-z'|E\right) + e^{jk_{zmn}|z-z'|} erfc\left(\frac{jk_{zmn}}{2E} + |z-z'|E\right)\right]$$

and

$$G_{p2}(\mathbf{r},\mathbf{r}') = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{e^{-j\mathbf{k}_{t00}\cdot\boldsymbol{\rho}_{mn}}}{8\pi R_{mn}} \times\left[e^{-jk_0 R_{mn}} erfc\left(R_{mn}E - \frac{jk}{2E}\right) + e^{jk_0 R_{mn}} erfc\left(R_{mn}E + \frac{jk}{2E}\right)\right]$$

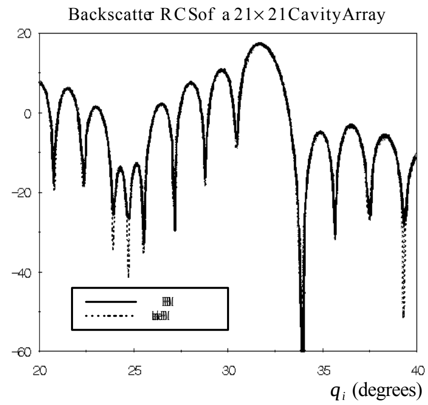


Fig. 2. Backscatter RCS of a 21×21 array of rectangular cavities recessed in a ground plane. For this computation, $T_x = 1l$, $T_y = 0.5l$, the cavity size is $0.9l \times 0.4l \times 0.1l$, $f_i = 20^\circ$, and the polarization is $E_f - E_f$ case

From the above it can be noticed that in the Ewald transformation method, the free space periodic Green's function is expressed as the sum of a modified spectral and a modified spatial series. The terms of these series have complementary error function in them and this leads to a series representation that exhibits a very rapid convergence rate. The parameter E controls the convergence rate. As E becomes larger, the spatial series converges faster, while the spectral series converges slower, and vice versa.

Now using this Green's function, we can compute the scattered fields on the fictitious boundary and the total fields on the boundary surface are the sum of these scattered fields, the incident fields and the fields reflected by a PEC ground plane introduced due to the equivalence principle.

In this way, the fields on the radiation boundary are updated, and this means the update of the residual term **U** in the equation (1). Through a few iterations of this procedure, the fields converge to an exact solution without interior resonance.

In this procedure, since the system matrix generated is sparse and banded as in conventional FEM and unchanged during the iterations, computational efficiency can be greatly enhanced.

Ⅲ. Numerical Results

To show the validity of the proposed method, we compared the result with that from FEBIM [8]. By practical necessity, the computed scattering pattern should be for a finite-size array. It was obtained by multiplying the scattering pattern for a single unit cell of the infinite array with the standard array factor. The results are shown in Fig. 2 and agree very well, which validates the infinite-array solution of the iterative FEM since the solution from FEBIM was also obtained from the same assumption. (Whether the assumption that the scattering pattern for a finite-size array can be approximated by the product of the scattering pattern for a single unit cell and the standard array factor is valid or not should be investigated, and for that matter, readers are referred to the reference [8].)

Ⅳ. CONCLUSION

In this paper, we showed that the iterative FEM can be successfully applied for the analysis of scattering from a three-dimensional doubly periodic cavity array. By virtue of the finite element method and the triangular prismatic volume elements, the proposed scheme is applicable to periodic arrays comprised of cavities having arbitrary shape and filled with inhomogeneous dielectrics. Comparing the result with that from available literature validates the proposed method

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