Efficient Calculation of the Green's Function for Multilayered Planar Periodic Structures

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***Abstract*—An efficient calculation scheme is presented for the periodic Green's function in planar multilayered structures. The proposed method is based on the static complex image conversion, and the Ewald sum technique and it converts the slowly convergent Green's function into the sum of two rapidly convergent series.**

***Index Terms*—Green's functions, nonhomogeneous media.**

I. INTRODUCTION

The periodic structures with planar multilayered media are widely used in constructing the frequency selective surfaces (FSS's). The method of moments (MoM) analysis of these structures gets complicated due to the slow convergence of the Green's function. Kipp [1] and Shubair [2] resolved this problem through the use of acceleration techniques of the free-space periodic Green's function [3], [4] combined with the complex image method [5] for the multilayered media.

This letter proposes another rapid method of calculation for the potential Green's functions of the periodic structure with layered media. In this method, most terms of the Green's function series are converted into the static image series in homogeneous medium, and are accelerated using the Ewald sum technique [6]. The resultant expression for the Green's function series has exponential convergence both in the space and in the spectral domain and only a small number of terms are required for its accurate calculation.

II. THEORY

The periodic Green's function in planar multilayered media can be written as follows [2]:

$$G_p = \frac{1}{ab}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\tilde{G}(k_{mn})e^{jk_{xm}(x-x')}e^{jk_{yn}(y-y')} \qquad (1)$$

$$k_{xm}=\frac{2m\pi}{a}-k_x^i, \quad k_{yn}=\frac{2n\pi}{b}-k_y^i, \quad k_{mn}=\sqrt{k_{xm}^2+k_{yn}^2}$$

where a, b are the periods of the structure and k_x^i, k_y^i are the wavenumbers of the incident plane wave in x, y directions, respectively. $\tilde{G}$ is the spectral domain Green's function for the nonperiodic, planar multilayered media.

Along the real k_{mn} axis, the spectral-domain Green's function has complex value only for $k_{mn} < k_{i,\max}(k_{i,\max} = \text{Max}\{k_i\}, k_i = \omega\sqrt{\mu_i\epsilon_i})$ and it is a real function for $k_{mn} > k_{i,\max}$. For this latter part, the spectral Green's function is approximated in terms of exponential functions as follows:

$$\tilde{G}(k_{mn}) = \frac{1}{k_{mn}}\sum_{i=1}^{N}C_i e^{s_i k_{mn}}, \qquad k_{mn} > k_{i,\max}. \qquad (2)$$

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TABLE I
NUMBER OF TERMS REQUIRED/RESULTANT % ERROR IN THE SCALAR POTENTIAL CALCULATION

x', y'	0.05a	0.15a	0.25a	0.35a	0.45a
0.05b	86/.22	91/.22	91/.23	91/.26	91/.29
0.15b	91/.21	92/.21	92/.22	92/.25	91/.29
0.25b	91/.22	92/.22	92/.23	91/.24	90/.28
0.35b	91/.26	92/.25	91/.24	92/.23	89/.25
0.45b	91/.30	91/.29	90/.26	89/.26	91/.04

There are several well-established numerical algorithms available for the exponential approximation of data, such as the Prony's method [7] or the generalized pencil-of-function (GPOF) method [8], which can be used in the above approximation. As compared with the conventional complex image method [5], the approximation of (2) is simpler since the approximated function is a real function defined on the real axis and no singularities are involved in the approximation range. Therefore, no path deformation [5] is required and the efficient, real arithmetic can be used in the numerical calculation. Substitution of (2) into (1) and some rearrangement of the resultant series yield the following result:

$$\begin{aligned} G_p = {}& \frac{1}{ab}\sum_{m}^{(k_{mn}<k_{i,\max})}\sum_{n}\left\{\tilde{G}(k_{mn}) - \frac{1}{k_{mn}}\sum_{i=1}^{N}C_i e^{s_i k_{mn}}\right\} \\ & \times e^{jk_{xm}(x-x')}e^{jk_{yn}(y-y')} \\ & + \sum_{i=1}^{N}C_i\frac{1}{ab}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{e^{s_i k_{mn}}}{k_{mn}}e^{jk_{xm}(x-x')} \\ & \times e^{jk_{yn}(y-y')}. \end{aligned} \qquad (3)$$

Now, each term of the second series corresponds to the electrostatic potential due to the two-dimensional periodic charge distribution with progressive phase shift, and can be calculated efficiently using the Ewald sum technique [6].

$$\begin{aligned} & \frac{1}{ab}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{e^{s_i k_{mn}}}{k_{mn}}e^{jk_{xm}(x-x')}e^{jk_{yn}(y-y')} \\ & = \frac{1}{2ab}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{1}{k_{mn}}\{e^{s_i k_{mn}}\text{erfc}(k_{mn}/2E + s_i E) \\ & \quad + e^{-s_i k_{mn}}\text{erfc}(k_{mn}/2E - s_i E)\}e^{jk_{xm}(x-x')}e^{jk_{yn}(y-y')} \\ & \quad + \frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{\text{erfc}(R_{mn,i}E)}{R_{mn,i}}e^{-jk_x^i ma}e^{-jk_y^i nb} \end{aligned} \qquad (4)$$

$$R_{mn,i} = \sqrt{(x-x'-ma)^2 + (y-y'-nb)^2 + s_i^2}$$

where erfc( ) is the complementary error function and E is the adjustable parameter in the Ewald sum method [6].

The two series on the right of (4) can be interpreted as the spectral- and the space-domain periodic Green's function weighted by the complementary error function, respectively. Since $\text{erfc}(x)$ converges at the rate of $\exp(-x^2)/x$, both the two series are rapidly convergent. Furthermore, the static image conversion in the present approach makes it possible to use the real-error function in the numerical calculation, which is computationally more efficient than the complex error function required for the Ewald sum of the dynamic image series [6].

TABLE II
AVERAGE NUMBER OF TERMS REQUIRED AND THE RESULTANT % ERROR IN THE CALCULATION OF THE POTENTIAL GREEN'S FUNCTION WITH VARIOUS SUBSTRATE/SUPERSTRATE PERMITTIVITIES. (a) ELECTRIC SCALAR POTENTIAL. (b) MAGNETIC VECTOR POTENTIAL

$(\varepsilon_{r1}, \varepsilon_{r2})$	(1,2)	(1,4)	(2,2	(2,4)	(4,4)
Terms	90.84	97.81	91.08	97.84	96.32
Error	0.239	0.008	0.633	0.004	0.033

(a)

$(\varepsilon_{r1}, \varepsilon_{r2})$	(1,2)	(1,4)	(2,2)	(2,4)	(4,4)
Terms	90.68	97.76	91.08	97.84	96.28
Error	0.206	0.008	0.463	0.004	0.045

(b)

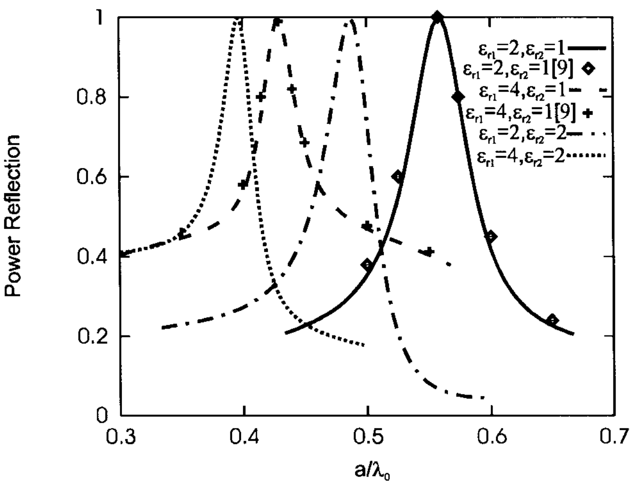

Fig. 1. Power reflection coefficient of a crossed-dipole FSS sandwiched between two dielectric substrates.

III. NUMERICAL RESULT

The proposed method of calculating the Green's function has been applied to investigate the frequency selective property of the periodic ($a = b = 10$ mm) crossed-dipole structure [1], [9] sandwiched between two dielectric layers. The geometrical details of this structure can be found in [1].

Tables I and II show the total number of terms used and the resultant percent error in the calculation of the Green's function by the proposed method. First, Table I shows the calculation results for the electric scalar potential Green's function at various observation points with the source point fixed at the center of a unit cell ($x' = 0.5a$, $y' = 0.5b$). This calculation has been conducted for a one-layer substrate case (dielectric constant 2, thickness 3 mm) at the frequency of 15 GHz with the incident wave $k_x^i = k_y^i = 0.5k_0$. The reference value for the error calculation has been obtained using the spectral-domain Green's function of (1) in which the slowly convergent, asymptotic series are subtracted and transformed into a rapidly convergent form in the space domain. In spite of this series acceleration technique, more than thousands of terms were generally required for this calculation. However, the calculation by the proposed method used only 90.84 terms on average with 0.239% average error when six static images were used with the Prony's method.

Similar calculations have been repeated for the vector and the scalar potential Green's functions for various superstrate/substrate permittivities and the averaged results are shown in Table II.

Finally, the crossed-dipole FSS has been analyzed using the MoM procedure with the Green's functions calculated by the present method. The Galerkin's method with the rectangular rooftop basis functions has been used in the MoM analysis. For the one-layer dielectric cases, the calculational results agree well with independent calculations in [9], which confirms the validity of the present method. Therefore, the proposed method can be used effectively in the analysis of complex periodic structures with layered media.

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