Spatially Focusing Electromagnetic Field in a Multi-Path Environment Using Time-reversal

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1. Introduction

Recently time reversal method is actively studied since it provides an efficient way to circumvent many negative effects of a wave propagation channel for many applications such as communication, radar, etc. In time-reversal of a narrow band signal, a phase conjugate or retrodirective array is used, which conjugates a received signal and then sends the modified signal back to the original source point. Hence the phase conjugate array works without any prior knowledge of the source's location or the characteristics of the surrounding environment, which indicates time-reversal is independent on the condition of the system operation such as a propagation channel, operation configuration (locations of transmitter & receiver), and so on. Following the aforementioned procedure a narrow pulse can be focused in both time and spatial domains. Especially, in a highly multi-path environment like a random media, a better performance, narrower pulse, can be expected since due to the multi-path, fields at adjacent points are statistically uncorrelated.

In this paper, it is theoretically shown that the spatial beamwidth of a time reversal pulse is related to a spatial correlation of the propagation channel [1]. Then it is demonstrated in a forest environment. Based on the observation, the stability of the time reversal pulse is simply mentioned.

2. Relationship between spatial correlation and beamwidth of time-reversal pulse

Spatial correlation of a propagation channel can be defined as, [1]

$$W_{ij}(\mathbf{r}_1,\mathbf{r}_2)=\langle \mathrm{E}_i^*(\mathbf{r}_1)\mathrm{E}_j(\mathbf{r}_2)\rangle \tag{1}$$

where E_i denotes the vector component of electric field in a given coordinate system, (for example, in Cartesian coordinate system subscript i can be x, y or z), and $*$, $\langle\ \rangle$ are complex conjugation, ensemble average, respectively. The fields at $\mathbf{r}_1$ and $\mathbf{r}_2$ are caused by a transmit antenna of the propagation channel. Also, spatial correlation coefficient can be defined as [2]

$$\rho_{ij}(\mathbf{r}_1,\mathbf{r}_2)=\frac{\langle \mathrm{E}_i^*(\mathbf{r}_1)\mathrm{E}_j(\mathbf{r}_2)\rangle}{\sqrt{\left|\mathrm{E}_i(\mathbf{r}_1)\right|^2\left|\mathrm{E}_j(\mathbf{r}_2)\right|^2}} \tag{2}$$

If $\mathbf{r}_1=\mathbf{r}_2$, (2) becomes unity. As the distance between two observation points increases, the spatial correlation generally decreases. In a multi-path environment field may be decorrelated very fast, and so the correlation exists within a short range, a few wavelengths.

In frequency domain, the following Lorentz reciprocity [3] relation holds

$$\int \mathbf{J}\cdot\mathbf{E}_p=\int \mathbf{J}_p\cdot\mathbf{E} \tag{3}$$

where $\mathbf{E}$ and $\mathbf{E}_p$ are electric field caused by current sources $\mathbf{J}$, $\mathbf{J}_p$ respectively. In this paper, $\mathbf{i}$ and $\mathbf{j}$ denote unit vectors in a given coordinate system, and elements of a phase conjugate array are all $\mathbf{k}$ directed. And $E_i(\mathbf{r},\mathbf{r}_p)$ denotes $\mathbf{i}$ component of the received electric field at $\mathbf{r}$ caused by a delta current source $\mathbf{J}_p = \mathbf{k}\delta(\mathbf{r}-\mathbf{r}_p)$ located at $\mathbf{r}_p$.

Now, let a delta source $\mathbf{J} = \mathbf{i}\delta(\mathbf{r}-\mathbf{r}_1)$ be located at $\mathbf{r}_1$, send signal to an element of the phase conjugate array located at $\mathbf{r}_p$, and phase conjugated signal is sent back to the source through the same medium. To achieve phase conjugation on an element of the array, a delta current source $\mathbf{k}E_k^*(\mathbf{r}_p,\mathbf{r}_1)\delta(\mathbf{r}-\mathbf{r}_p) = \mathbf{k}E_i^*(\mathbf{r}_1,\mathbf{r}_p)\delta(\mathbf{r}-\mathbf{r}_p)$ at $\mathbf{r}_p$ is placed. The above equality $E_i(\mathbf{r}_1,\mathbf{r}_p) = E_k(\mathbf{r}_p,\mathbf{r}_1)$ can be obtained by substituting $\mathbf{J} = \mathbf{i}\delta(\mathbf{r}-\mathbf{r}_1)$ and $\mathbf{J}_p = \mathbf{k}\delta(\mathbf{r}-\mathbf{r}_p)$ into (3). Then the $\mathbf{j}$ component of the received field at $\mathbf{r}_2$ after phase conjugation is given by

$$E_i^*(\mathbf{r}_1,\mathbf{r}_p)E_j(\mathbf{r}_2,\mathbf{r}_p) \tag{4}$$

With N elements in the phase conjugate array, the $\mathbf{j}$ component of the resultant electric field at $\mathbf{r}_2$ after phase conjugation is given by

$$\sum_{n=1}^{N} E_i^*(\mathbf{r}_1,\mathbf{r}_{pn})E_j(\mathbf{r}_2,\mathbf{r}_{pn}) \tag{5}$$

where $\mathbf{r}_{pn}$ indicates the location of nth element of the phase conjugate array.

Since all elements are located in the same multi-path environment, (5) divided by N is an unbiased statistical estimator of (1), where two field components correlated are caused by $\mathbf{k}$ directed delta source. Hence if the number of elements of the phase conjugate array increases, the pattern of the resulting time reversal spatial pulse approaches to the spatial correlation of the propagation channel. Based on the observation, two important properties of time-reversal pulse can be induced: 1) the lowest limit of the spatial beamwidth of time reversal pulse equals to that of the spatial correlation of the propagation channel. 2) If the number of antennas in the time-reversal array increases, a stable time reversal pulse can be obtained, whose pattern is affected in a small amount by a sounding channel. To minimize number of antennas, elements in the array must be separated enough so that fields generated by each element are uncorrelated.

3. Simulations Using a Physics Based Fractal Tree Model

In order to demonstrate the feasibility of applying phase conjugate array to practical problems, a forest channel model, developed at the University of Michigan's Radiation Laboratory is used to calculate characteristics of a highly scattering, and inhomogeneous environment [3].

The physical scenario for simulations is shown in Figure 1. A phase conjugate array located inside the forest receives the field, and other antenna is located at the elevation angle 10 degrees and the azimuth angle 90 degrees. In this situation, the spatial correlation can be calculated in the unit of angle, not distance, since electric fields are measured in the far field. Vertical and horizontal components in a spherical coordinate are illustrated in Figure 2. $\mathbf{v}$ and $\mathbf{h}$ in Figure 2 are unit vectors in the respective direction. Frequency was set to be 10 GHz.

In figure 3, absolute value of spatial correlation coefficient (2) with one delta current source located inside the forest was estimated by ensemble averaging over 10 realizations of randomly generated forest, where $\mathbf{i},\mathbf{j} = \mathbf{h}$ and $\mathbf{k} = \mathbf{x},\mathbf{y},\mathbf{z}$. Figure 4 shows the same estimate, but for this case it was obtained by averaging over 200 realizations. And in figure 5, the phase of the estimate shown in figure 4 is plotted. As the sample (i.e. realization) number increases, more accurate estimate is obtained.

In figure 6 normalized $\mathbf{h}$ component of electric field pattern returned from phase conjugate array with one element is plotted. Figure 7 shows the same normalized electric field pattern but for this case 5 array elements are used separated by one wavelength. In figure 8, the number of array elements, with the same spacing between the elements, increases to 10. In short, $\mathbf{i},\mathbf{j}=\mathbf{h}$ in (5), and all elements of the array are $\mathbf{k}(=\mathbf{x},\mathbf{y},\mathbf{z})$ directed. It can be observed that with increasing number of elements in one realization, the normalized returned field pattern from the phase conjugate array resembles the spatial correlation pattern shown in Figure 4, and the side lobe level is reduced fast as expected. Analog to time ergodicity in the random process theory, this observation might be called spatial ergodicity in a multi-path environment.

In figure 8, two advantages of the explained phase conjugation process can be observed. First, phase conjugation intensifies electric field focused at the original source point. If an arbitrary array were used, focusing wouldn't appear but random fluctuation would be observed depending on a specific realization. Second, narrow beamwidth can be achieved with relatively small number of elements. Approximately 0.5 degrees of beamwidth is obtained by using 10 elements. To achieve this beamwidth from a broadside array, about 240 phased elements spaced 1 wavelength apart are needed in free space [4]. However, in one realization, nonzero correlation between fields generated by each element, can degrade the quality of focusing. For example, figure 8 shows, compared to figure 3, higher sidelobe level to some degree, due to correlation.

Meanwhile, fields from $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ directed elements of the phase conjugate array are not much correlated as shown in figure 5, which plots the phase of the spatial correlation. This implies that averaging over polarization of the elements in array can be exploited, besides the averaging over spatially many elements in the array.

4. Conclusions

In this paper, it was demonstrated that a field from a large phase conjugate array is a statistical estimate of the spatial correlation of the wave propagation channel. In many situations the spatial correlation is known, and decays very fast. Hence the focusing of electromagnetic field within a few wavelength distance can be achieved by time reversal as seen in simulations, and the lowest limit of the beamwidth of the focused pulse can be estimated without any effort.

References:

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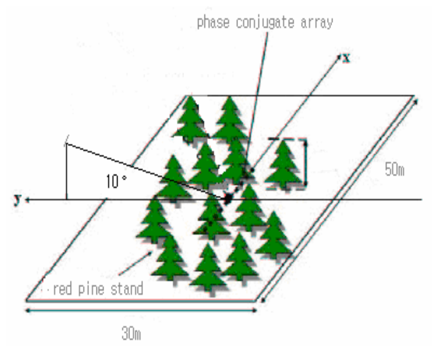

Fig. 1 simulated forest scenario

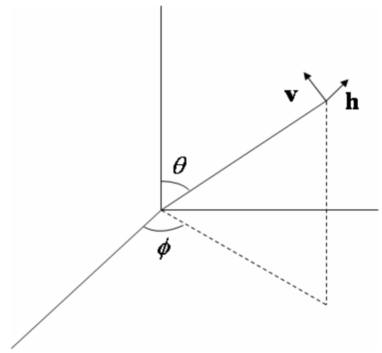

Fig. 2 coordinate suitable for far field correlation

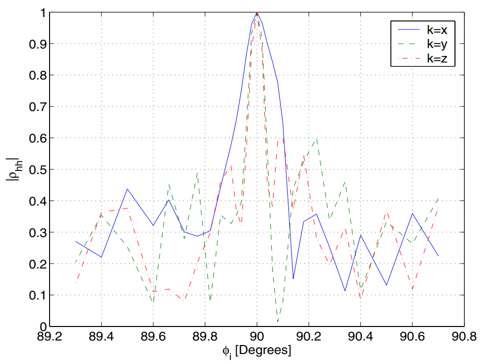

Fig. 3 estimate of spatial correlation $|\rho_{hh}|$ ensemble averaged over 10 realizations

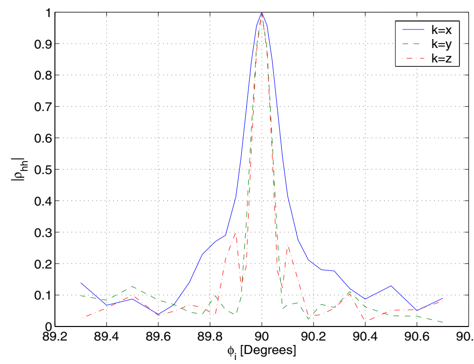

Fig. 4 estimate of spatial correlation $|\rho_{hh}|$ ensemble averaged over 200 realizations

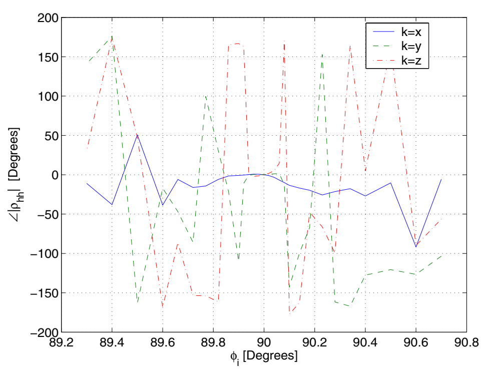

Fig. 5 phase of the estimate in Fig. 4

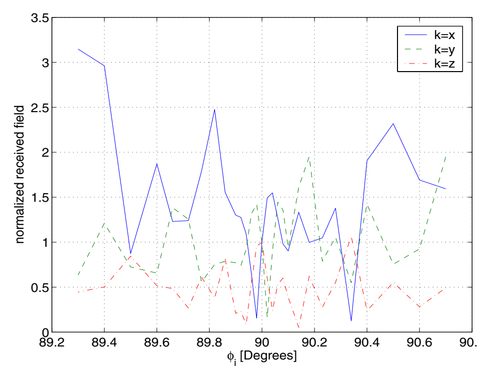

Fig. 6 normalized received field pattern, N=1

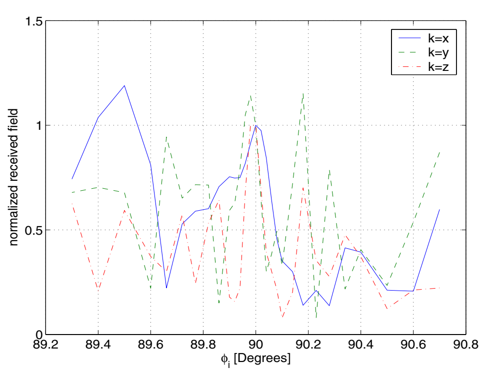

Fig. 6 normalized received field pattern, N=5

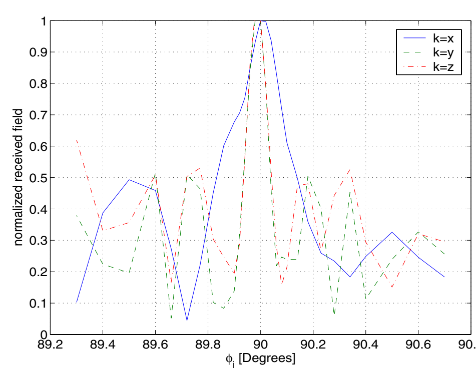

Fig. 7 normalized received field pattern, N=10