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A NEW DERIVATION FOR THE COEFFICIENTS OF THE SCATTERING DYADIC GREEN'S FUNCTION IN SPHERICALLY LAYERED MEDIA

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ABSTRACT: *The dyadic Green's function in spherically layered media is considered by assuming that source and field points can be located anywhere. After manipulations from the expression of the dyadic Green's function based on the reflections and transmissions of the scalar waves, the dyadic Green's function in terms of the spherical vector wave functions is newly constructed and also the coefficients of the scattering dyadic Green's function are derived in a simple form. © 2007 Wiley Periodicals, Inc. Microwave Opt Technol Lett 49: 1142–1143, 2007; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.22374*

1. INTRODUCTION

The dyadic Green's function (DGF) has been an interesting topic in electromagnetic theory. Although most problems can be solved without the use of DGFs, the symbolic simplicity with which they could be used to express relationships makes the formulations of many problems simpler and more compact. The DGFs for simple spherical geometries were introduced earlier [1], and a general expression of DGF for a spherically arbitrary layered medium was constructed [2]. In both works, the DGFs in terms of the spherical vector wave functions (VWFs) were first expressed in each layer using the method of scattering superposition, and then the unknown coefficients within them were solved based on the coupling
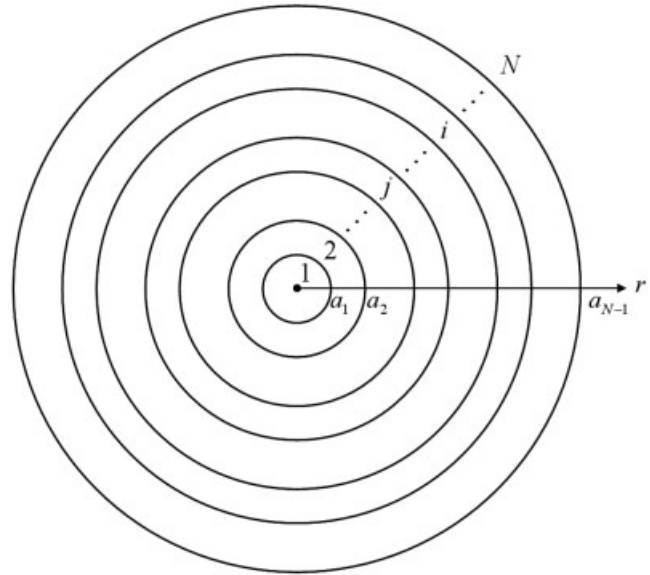

Figure 1 Geometry of spherical of *N*-layered medium

recurrence matrix equations by imposing boundary conditions at all interfaces. In this article, starting from the DGF formulated by Chew [3], which is based on the reflections and transmissions of the scalar waves characterized by Debye potentials, a simple and direct way of deriving the coefficients of the scattering DGF when expressed in terms of spherical VWFs is presented. The time-dependent $\exp(-i\omega t)$ is assumed for the field quantities throughout the paper.

2. GENERAL EXPRESSION OF DGF

The spherical concentric *N*-layered medium under consideration is shown in Figure 1. The complex permittivity and permeability of *i*th layer are described by ε_i and μ_i, respectively. The DGF of electric type for a spherically layered medium is given in Ref. 3 (Eq. 7.4.32).

$$\overline{\mathbf{G}}_e^{(ij)}(\mathbf{r}, \mathbf{r}') = ik_j \sum_{n=0}^{\infty} \frac{1}{n(n+1)} [\overline{\mathbf{m}}_n(\mathbf{r}, \mathbf{r}') + \overline{\mathbf{n}}_n(\mathbf{r}, \mathbf{r}')] \quad (\mathbf{r} \neq \mathbf{r}') \quad (1)$$

where $k_j = k_0 \sqrt{\varepsilon_j \mu_j}$, and it is assumed that the source point $\mathbf{r}'$ is in the layer j and the field point $\mathbf{r}$ is in the layer i ($i, j = 1, 2, \ldots, N$). $\overline{\mathbf{m}}_n$ and $\overline{\mathbf{n}}_n$ in Eq. (1) are the quantities that are proportional to the Debye potentials and defined by [3]:

$$\overline{\mathbf{m}}_n(\mathbf{r}, \mathbf{r}') = (\nabla \times \mathbf{r})(\nabla' \times \mathbf{r}') F_n^{\text{TE}}(r, r') A_n(\theta, \phi; \theta', \phi') \quad (2)$$

$$\overline{\mathbf{n}}_n(\mathbf{r}, \mathbf{r}') = \left(\frac{\nabla \times \nabla \times \mathbf{r}}{-i\omega\varepsilon_i} \right) \left(\frac{\nabla' \times \nabla' \times \mathbf{r}'}{i\omega\mu_j} \right) F_n^{\text{TM}}(r, r') A_n(\theta, \phi; \theta', \phi')$$

$$(3)$$

where

$$F_n(r, r') = \begin{cases} [h_n^{(1)}(k_j r) + j_n(k_j r)\tilde{R}_{j,j+1}]\tilde{M}_j[j_n(k_j r') + h_n^{(1)}(k_j r')\tilde{R}_{j,j-1}] & (r > r'), \ i = j \\ [j_n(k_j r) + h_n^{(1)}(k_j r)\tilde{R}_{j,j-1}]\tilde{M}_j[h_n^{(1)}(k_j r') + j_n(k_j r')\tilde{R}_{j,j+1}] & (r < r'), \ i = j \\ [h_n^{(1)}(k_i r) + j_n(k_i r)\tilde{R}_{i,i+1}]\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j[j_n(k_j r') + h_n^{(1)}(k_j r')\tilde{R}_{j,j-1}], & i > j \\ [j_n(k_i r) + h_n^{(1)}(k_i r)\tilde{R}_{i,i-1}]\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j[h_n^{(1)}(k_j r') + j_n(k_j r')\tilde{R}_{j,j+1}], & i < j \end{cases} \quad (4)$$

$$A_n(\theta,\phi;\theta',\phi') = \sum_{m=-n}^{n} Y_{nm}(\theta,\phi) Y_{n,-m}(\theta',\phi') \quad (5)$$

$\tilde{R}$, $\tilde{T}$, and $\tilde{M}_j = (1 - \tilde{R}_{j,j+1}\tilde{R}_{j,j-1})^{-1}$ in Eq. (4) are generalized reflection/transmission coefficients for either TE or TM waves and a factor accounting for multiple reflections in the layer j, respectively, and Y_{nm} in Eq. (5) is a spherical harmonic function [3]. Using that $\tilde{M}_j = 1 + (\tilde{M}_j - 1) = 1 + \tilde{R}_{j,j+1}\tilde{R}_{j,j-1}\tilde{M}_j$, F_n can be separated into two terms, an unbounded (direct) term a scattering one:

$$F_n(r, r') = F_{nd}(r, r')\delta_{ij} + F_{ns}(r, r') \quad (6)$$

where δ_{ij} is the Kronecker delta function and

$$F_{nd}(r, r') = \begin{cases} h_n^{(1)}(k_j r) j_n(k_j r'), & r > r' \\ j_n(k_j r) h_n^{(1)}(k_j r'), & r < r' \end{cases} \quad (7)$$

$$F_{ns}(r, r') = A_n h_n^{(1)}(k_i r) j_n(k_j r') + B_n h_n^{(1)}(k_i r) h_n^{(1)}(k_j r')$$
$$+ C_n j_n(k_i r) j_n(k_j r') + D_n j_n(k_i r) h_n^{(1)}(k_j r') \quad (8)$$

A_n to D_n in Eq. (8) are the coefficients of the scattering DGF for either TE and TM waves and can be expressed in terms of $\tilde{R}$, $\tilde{T}$, and $\tilde{M}$, which will be explained later in detail. By substituting (2) and (3) back into (1), after some manipulations, $\overline{\mathbf{G}}_e^{(ij)}(\mathbf{r}, \mathbf{r}')$ also can be separated into two terms and constructed in terms of the spherical VWFs, $\mathbf{M}$ and $\mathbf{N}$ [1]:

$$\overline{\mathbf{G}}_e^{(ij)}(\mathbf{r}, \mathbf{r}') = \overline{\mathbf{G}}_{e0}(\mathbf{r}, \mathbf{r}')\delta_{ij} + \overline{\mathbf{G}}_{es}^{(ij)}(\mathbf{r}, \mathbf{r}') \quad (9)$$

$$\overline{\mathbf{G}}_{e0}(\mathbf{r}, \mathbf{r}') = \frac{ik_j}{4\pi} \sum_{n=1}^{\infty}\sum_{m=0}^{n} (2 - \delta_{m0})\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}$$
$$\times \begin{cases} \mathbf{M}_{omn}^{(1)}(k_j)\mathbf{M}'_{omn}(k_j) + \mathbf{N}_{omn}^{(1)}(k_j)\mathbf{N}'_{omn}(k_j), & r > r' \\ \mathbf{M}_{omn}(k_j)\mathbf{M}_{omn}^{'(1)}(k_j) + \mathbf{N}_{omn}(k_j)\mathbf{N}_{omn}^{'(1)}(k_j), & r < r' \end{cases} \quad (10)$$

$$\overline{\mathbf{G}}_{es}^{(ij)}(\mathbf{r}, \mathbf{r}') = \frac{ik_j}{4\pi} \sum_{n=1}^{\infty}\sum_{m=0}^{n} (2 - \delta_{m0})\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}$$
$$\times [(1 - \delta_{i1})\mathbf{M}_{omn}^{(1)}(k_i)\{(1 - \delta_{jN}) A_n^{\mathrm{TE}}\mathbf{M}'_{omn}(k_j)$$
$$+ (1 - \delta_{j1}) B_n^{\mathrm{TE}}\mathbf{M}_{omn}^{'(1)}(k_j)\} + (1 - \delta_{iN})\mathbf{M}_{omn}^e(k_i)\{(1$$
$$- \delta_{jN})C_n^{\mathrm{TE}}\mathbf{M}'_{omn}(k_j) + (1 - \delta_{j1}) D_n^{\mathrm{TE}}\mathbf{M}_{omn}^{'(1)}(k_{j1})\}$$
$$+ (1 - \delta_{i1})\mathbf{N}_{omn}^{(1)}(k_i)\{(1 - \delta_{jN}) A_n^{\mathrm{TM}}\mathbf{N}'_{omn}(k_j)$$
$$+ (1 - \delta_{j1}) B_n^{\mathrm{TM}}\mathbf{N}_{omn}^{'(1)}(k_j)\}K^{\mathrm{TM}} + (1 - \delta_{iN})\mathbf{N}_{omn}^e(k_i)$$
$$\times \{(1 - \delta_{jN})C_n^{\mathrm{TM}}\mathbf{N}'_{omn}(k_j) + (1 - \delta_{j1}) D_n^{\mathrm{TM}}\mathbf{N}_{omn}^{'(1)}(k_j)\}K^{\mathrm{TM}}] \quad (11)$$

where $K^{\mathrm{TM}} = \sqrt{(\varepsilon_j\mu_i)/(\varepsilon_i\mu_j)}$ is a constant for TM modes and $\overline{\mathbf{G}}_{e0}(\mathbf{r}, \mathbf{r}')$ represents the contribution of the direct waves while $\overline{\mathbf{G}}_{es}^{(ij)}(\mathbf{r}, \mathbf{r}')$ describes that of the multiple reflections and transmissions of waves in the presence of a layered medium.

TABLE 1 Coefficients of the Scattering DGF Assuming That the Source Point is in Region j and the Field Point is in Region i

	$i = j$	$i > j$	$i < j$
A_n	$\tilde{R}_{j,j-1}\tilde{M}_j\tilde{R}_{j,j+1}$	$\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j$	$\tilde{R}_{i,i-1}\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j\tilde{R}_{j,j+1}$
B_n	$\tilde{M}_j\tilde{R}_{j,j-1}$	$\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j\tilde{R}_{j,j-1}$	$\tilde{R}_{i,i-1}\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j$
C_n	$\tilde{M}_j\tilde{R}_{j,j+1}$	$\tilde{R}_{i,i+1}\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j$	$\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j\tilde{R}_{j,j+1}$
D_n	$\tilde{R}_{j,j+1}\tilde{M}_j\tilde{R}_{j,j-1}$	$\tilde{R}_{i,i+1}\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j\tilde{R}_{j,j-1}$	$\tilde{M}_i\tilde{T}_{ji}\tilde{M}_j$

3. COEFFICIENTS OF SCATTERING DGF

Since the unbounded DGF $\overline{\mathbf{G}}_{e0}(\mathbf{r}, \mathbf{r}')$ is usually evaluated analytically from the spatial representation $\overline{\mathbf{G}}_{e0}(\mathbf{r}, \mathbf{r}') = (\overline{\mathbf{I}} + \nabla\nabla/k^2) G_0(\mathbf{r}, \mathbf{r}')$, where $G_0(\mathbf{r}, \mathbf{r}')$ is the scalar Green's function, only the scattering DGF and its coefficients are considered here. In deriving Eq. (8), four coefficients can be expressed as shown in Table 1, and these are easily calculated using the single-interface reflection/transmission coefficients. Physically, the coefficients A_n and B_n represent the outgoing waves while C_n and D_n represent the standing waves in the field region i. In addition, A_n and C_n represent the waves whose initial reflections or transmissions are occurred at the outer interface while B_n and D_n represent the waves whose initial reflections or transmissions are occurred at the inner interface of the source region j.

4. CONCLUSION

The dyadic Green's function has been newly constructed in terms of the spherical vector wave functions based on the reflections and transmissions of the scalar waves characterized by Debye potentials. The coefficients of the scattering dyadic Green's function also have been derived in this process, which is compact, intuitively derivable and suitable to elucidate the physics better.

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